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Realization of PID controls by fuzzy control methods

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Abstract

This paper shows that PID controllers can be realized by fuzzy control methods of "product–sum-gravity method" and "simplified fuzzy reasoning method". PID controllers, however, cannot be constructed by min–max-gravity method known as Mamdani's fuzzy reasoning method. Furthermore, extrapolative reasoning can be executed by the product–sum-gravity method and simplified fuzzy reasoning method by extending membership functions of antecedent parts of fuzzy rules.

Keywords: Product–sum-gravity method; Simplified fuzzy reasoning method; Min–max-gravity method; PID control; Extrapolative reasoning

1. Introduction

PID controllers are widely used as simple and effective controllers. In this paper, it is shown that PID controllers can be realized by fuzzy control methods called "product–sum-gravity method" and "simplified fuzzy reasoning method". Therefore, PID controls are shown to be a special case of fuzzy controls. PID controllers, however, cannot be constructed by min–max-gravity method which is known as Mamdani's fuzzy reasoning method.

Moreover, extrapolative reasoning can be executed by the product–sum-gravity method and simplified fuzzy reasoning method by extending the range of membership functions of antecedent parts of fuzzy rules from [0, 1] to $(-\infty, \infty)$.

2. Fuzzy reasoning methods of min–max–gravity method, product–sum-gravity method and simplified fuzzy reasoning method

We shall consider the following multiple fuzzy reasoning form:

$$\begin{array}{lll} \text{Rule 1:} & A1 \text{ and } B1 & \Rightarrow C1 \\ \text{Rule 2:} & A2 \text{ and } B2 & \Rightarrow C2 \\ & \vdots & \\ \text{Rule } n\text{:} & An \text{ and } Bn & \Rightarrow Cn \\ \text{Fact:} & x_0 \text{ and } y_0 & \\ \hline \text{Cons:} & & C' \end{array} \qquad (1)$$

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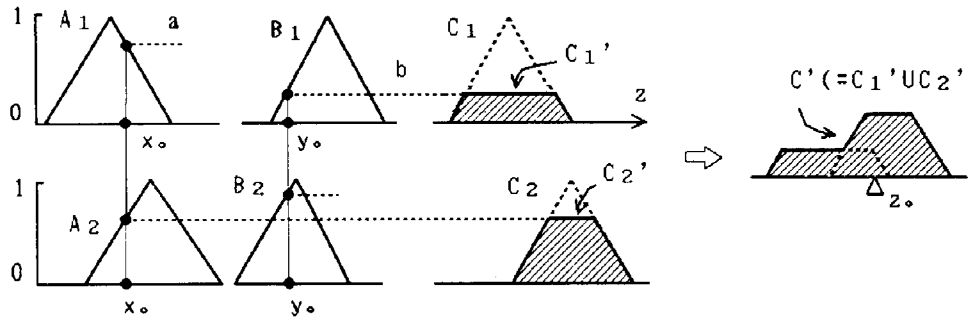


Fig. 1. Min–max-gravity method.

where Ai is a fuzzy set in X; Bi in Y; and Ci in Z and $x_0 \in X$, $y_0 \in Y$.

At first, we shall explain the case of Mamdani's fuzzy reasoning method [2] called *min–max-gravity method* (see Fig. 1). The inference result Ci' ($i = 1, \ldots, n$) which is inferred from the fact ⌈x_0 and y_0⌋ and the fuzzy rule ⌈Ai and $Bi \Rightarrow Ci$⌋ is given as

$$\mu_{Ci'}(z) = \mu_{Ai}(x_0) \wedge \mu_{Bi}(y_0) \wedge \mu_{Ci}(z), \tag{2}$$

where $\wedge$ stands for min.

The final consequence C' of (1) is aggregated by taking the union ($\cup$) of $C1', C2', \ldots, Cn'$ obtained in (2). Namely,

$$C' = C1' \cup C2' \cup \cdots \cup Cn', \tag{3}$$

that is,

$$\mu_{C'}(z) = \mu_{C1'}(z) \vee \cdots \vee \mu_{Cn'}(z), \tag{4}$$

where $\vee$ stands for max.

The representative point z_0 for the resulting fuzzy set C' is obtained as the center of gravity of C':

$$z_0 = \frac{\int z \cdot \mu_{C'}(z)\,\mathrm{d}z}{\int \mu_{C'}(z)\,\mathrm{d}z}. \tag{5}$$

This fuzzy reasoning method is known as Mamdani's method and called "*min–max-gravity method*". Most of the existing fuzzy logic controllers are based on the method. But it is pointed out in [4, 5] that this method does not necessarily fit our intuition.

We shall next show a fuzzy reasoning method called "*product–sum-gravity method*" [4, 5] which can be obtained by replacing min by algebraic product, and max by sum in the min–max-gravity method in (2) and (4). Namely, the product–sum-gravity method is defined as in Fig. 2.

The inference result Ci' is given as

$$\mu_{Ci'}(z) = \mu_{Ai}(x_0) \cdot \mu_{Bi}(y_0) \cdot \mu_{Ci}(z), \tag{6}$$

where $\cdot$ stands for algebraic product. The final consequence C' of (1) is aggregated by taking the sum ($+$) of $C1', C2', \ldots, Cn'$ obtained in (6). Namely,

$$\begin{aligned} C' &= C1' + C2' + \cdots + Cn', \\ \mu_{C'}(z) &= \mu_{C1'}(z) + \cdots + \mu_{Cn'}(z), \end{aligned} \tag{7}$$

where $+$ stands for sum.

The representative point z_0 for the fuzzy set C' is obtained as the center of gravity of C' as in (5).

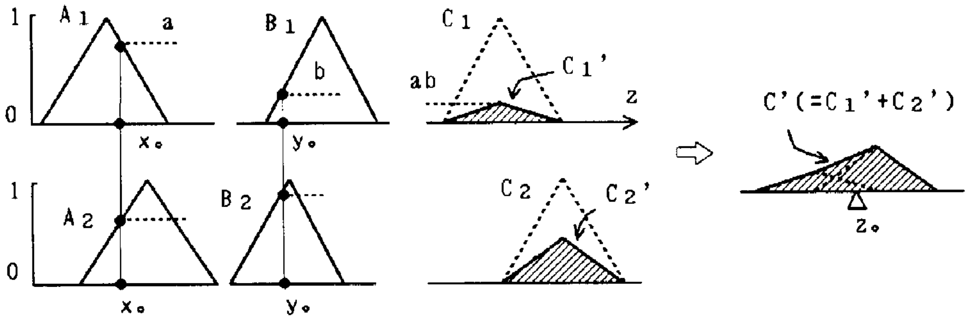


Fig. 2. Product–sum-gravity method.

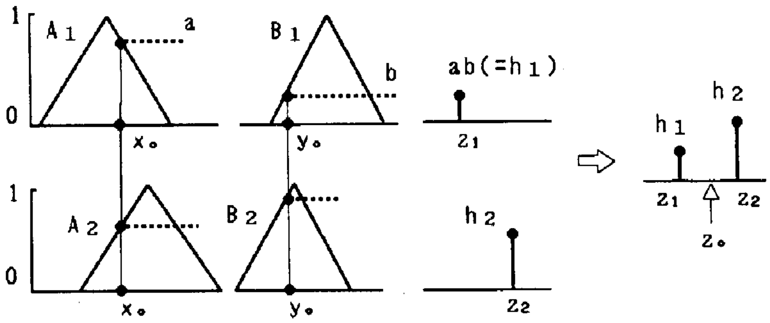


Fig. 3. Simplified fuzzy reasoning method.

It should be noted that the resulting fuzzy set C' sometimes exceeds 1 because of the use of sum operation in (7). To avoid such a fact, we can use arithmetic mean [4] instead of using the sum operation. However, the same center of gravity can be obtained even if we use sum or arithmetic mean in the aggregation.

As a special case of the product–sum-gravity method, we can give a *simplified fuzzy reasoning method* [1] for the following fuzzy reasoning form:

$$\begin{array}{ll} \text{Rule 1:} & A1 \text{ and } B1 \Rightarrow z1 \\ \text{Rule 2:} & A2 \text{ and } B2 \Rightarrow z2 \\ & \vdots \\ \text{Rule } n\text{:} & An \text{ and } Bn \Rightarrow zn \\ \text{Fact:} & x_0 \text{ and } y_0 \\ \hline \text{Cons:} & \qquad\qquad z_0 \end{array} \tag{8}$$

in which the consequent parts of the fuzzy rules are not fuzzy sets but real numbers $z1, z2, \ldots, zn$ in Z.

The consequence z_0 by the simplified fuzzy reasoning method is inferred as follows (see Fig. 3). The degree of fitness of the fact ⌈x_0 and y_0⌋ to the antecedent part ⌈Ai and Bi⌋ is given as

$$hi = \mu_{Ai}(x_0) \cdot \mu_{Bi}(y_0). \tag{9}$$

The degree of fitness, hi, may be regarded as the degree with which zi is obtained. Therefore, the final consequence z_0 of (8) is obtained as the weighted average of zi by the degree hi. Namely,

$$z_0 = \frac{h1 \cdot z1 + h2 \cdot z2 + \cdots + hn \cdot zn}{h1 + h2 + \cdots + hn}. \tag{10}$$

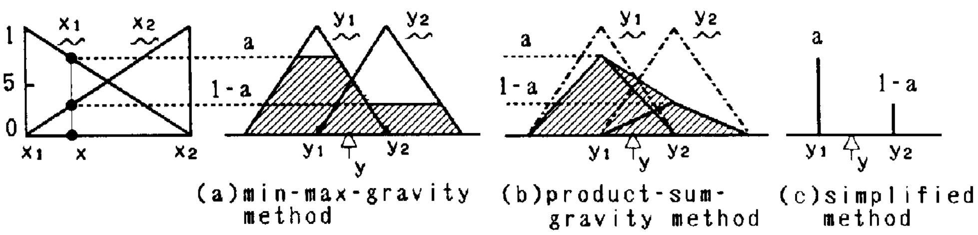


Fig. 4. Reasoning process of (11) by three fuzzy reasoning methods.

Note that the simplified fuzzy reasoning method is regarded as a special case of product–sum-gravity method. In fact, when the fuzzy sets $C1, C2, \ldots, Cn$ in the consequent part of (1) are all same in size, the product–sum-gravity method is reduced to the simplified fuzzy reasoning method. But the simplified fuzzy reasoning method is not a special case of min–max-gravity method.

We shall next discuss and compare the inference results by these fuzzy reasoning methods of min–max-gravity method, product–sum-gravity method and simplified fuzzy reasoning method.

Consider the following simple fuzzy reasoning form (see Fig. 4):

$$\begin{array}{ll} \text{Rule 1:} & \underset{\sim}{x1} \Rightarrow \underset{\sim}{y1} \\ \text{Rule 2:} & \underset{\sim}{x2} \Rightarrow \underset{\sim}{y2} \\ \text{Fact:} & x \\ \hline \text{Cons:} & \qquad y \end{array} \tag{11}$$

where fuzzy sets $\underset{\sim}{x1}$ (about $x1$) and $\underset{\sim}{x2}$ are of right-angled triangular type with the same width and intersect at the height 0.5. Fuzzy sets $\underset{\sim}{y1}$ and $\underset{\sim}{y2}$ are of isosceles triangular type with the same width and intersect at 0.5.

The inference result y at x of (11) by min–max-gravity method is obtained as follows [3, 7]:

$$y = \frac{a(3-a)y1 + (1-a)(2+a)y^2}{2(1+a+a^2)}, \tag{12}$$

where

$$a = \mu_{\underset{\sim}{x1}}(x) = \frac{x2 - x}{x2 - x1}.$$

On the other hand, product–sum-gravity method and simplified fuzzy reasoning method give such an inference result as

$$y = ay1 + (1-a)y2. \tag{13}$$

It is found from (12) and Fig. 5 that the min–max-gravity method gives a complicated inference result of nonlinear form for a simple fuzzy reasoning form (11), although product–sum-gravity method and simplified fuzzy reasoning method give a very simple inference result of linear form.

3. Realization of PID controllers by fuzzy reasoning methods

As is well known, a control action u of *PID controller* is given in the form of the linear combination of the error e, the change in error Δe and the integral $\int e\,dt$, namely,

$$u = \alpha e + \beta \Delta e + \gamma \int e\,dt, \tag{14}$$

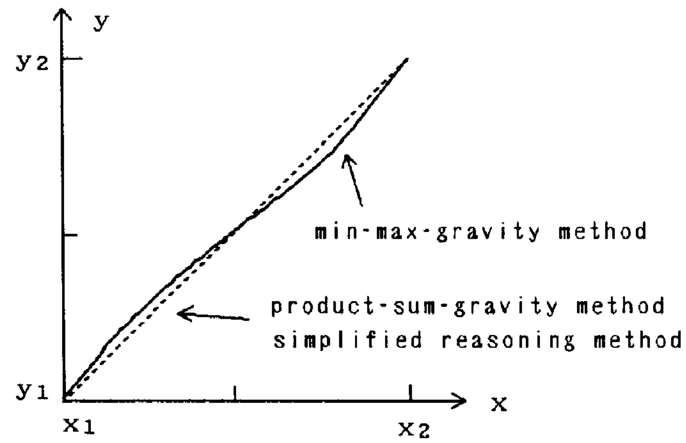


Fig. 5. Inference results for (11).

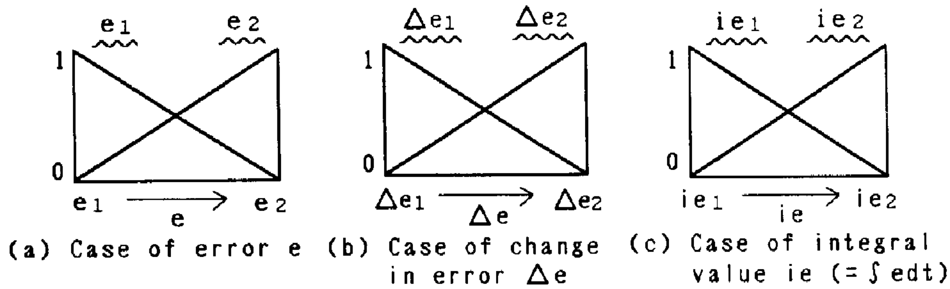


Fig. 6. Fuzzy sets of error e, change in error Δe and integral value ie.

where α is the propositional coefficient, β is the derivative coefficient, and γ is the integral coefficient for e.

For simplicity, we shall first discuss the realization of *PD controller* by fuzzy control methods of simplified fuzzy reasoning method and product–sum-gravity method, but not by min–max-gravity method. PD controller is given as

$$u = \alpha e + \beta \Delta e. \tag{15}$$

Let $e1$ and $e2$ be the minimal and maximal values of the possible error e, and let $\Delta e1$ and $\Delta e2$ be the minimal and maximal values of Δe, that is

$$e1 \leqslant e \leqslant e2; \qquad \Delta e1 \leqslant \Delta e \leqslant \Delta e2. \tag{16}$$

We shall define fuzzy sets $\underset{\sim}{e1}$ and $\underset{\sim}{e2}$ for the error e as in Fig. 6(a). Similarly, fuzzy sets $\underset{\sim}{\Delta e1}$ and $\underset{\sim}{\Delta e2}$ for the change in error Δe are given in Fig. 6(b).

We can give fuzzy control rules of simplified fuzzy reasoning method which realizes PD controller as follows:

$$\begin{array}{ll} \text{Rule 1:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e1} \Rightarrow u1 \\ \text{Rule 2:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e2} \Rightarrow u2 \\ \text{Rule 3:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e1} \Rightarrow u3 \\ \text{Rule 4:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e2} \Rightarrow u4 \\ \text{Fact:} & e \text{ and } \Delta e \\ \hline \text{Cons:} & \qquad\qquad u \end{array} \tag{17}$$

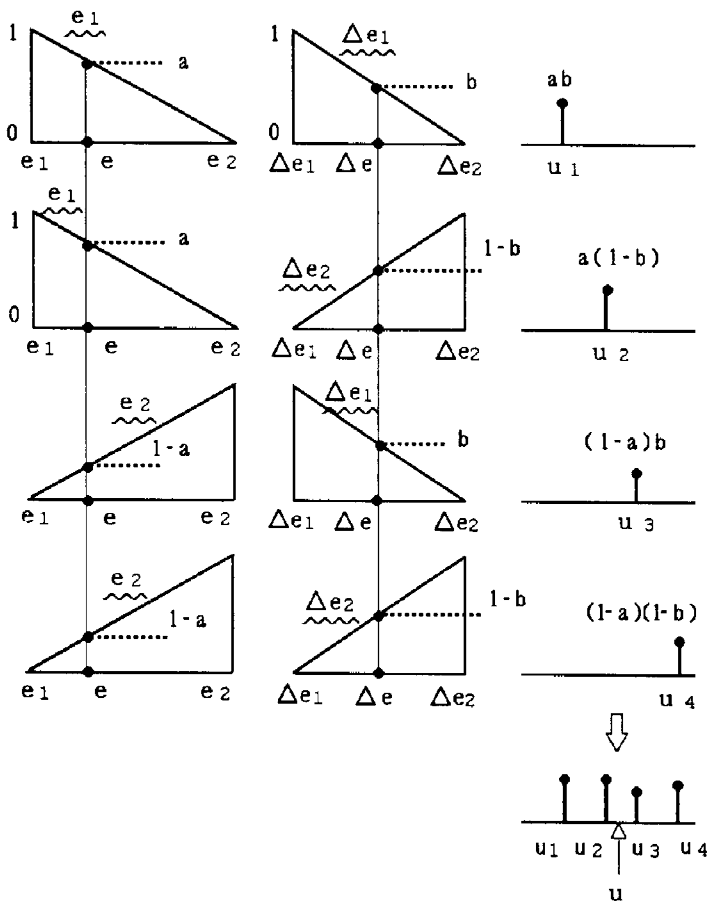


Fig. 7. Illustration of fuzzy reasoning of (17) by simplified reasoning method.

where $u1$, $u2$, $u3$ and $u4$ are real numbers such that

$$\begin{aligned} u1 &= \alpha e1 + \beta \Delta e1 \\ u2 &= \alpha e1 + \beta \Delta e2 \\ u3 &= \alpha e2 + \beta \Delta e1 \\ u4 &= \alpha e2 + \beta \Delta e2 \end{aligned} \tag{18}$$

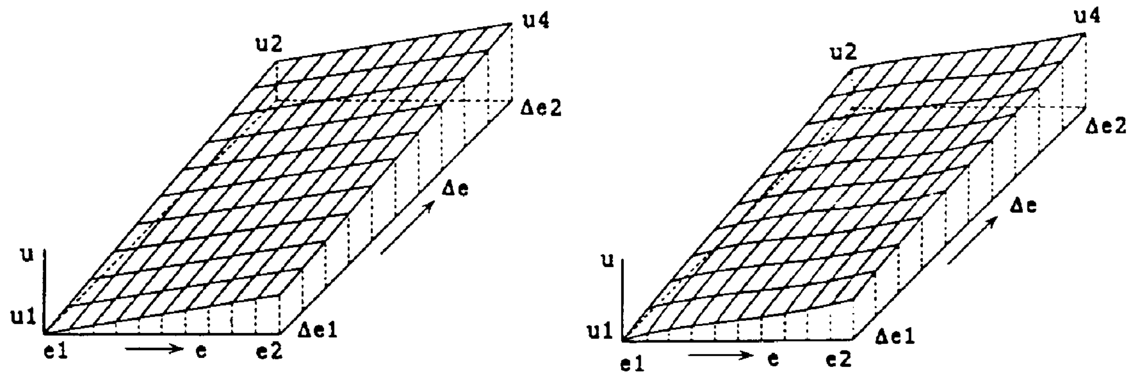
which are the values of (15) at points $(e1, \Delta e1)$, $(e1, \Delta e2)$, $(e2, \Delta e1)$ and $(e2, \Delta e2)$.

Then we can obtain an inference result u of (17) from (9) and (10) in the following (see Fig. 7):

$$\begin{aligned} u &= \frac{abu1 + a(1-b)u2 + (1-a)bu3 + (1-a)(1-b)u4}{ab + a(1-b) + (1-a)b + (1-a)(1-b)} \\ &= abu1 + a(1-b)u2 + (1-a)bu3 + (1-a)(1-b)u4. \end{aligned} \tag{19}$$

It is noted that the denominator of the above equation is equal to 1. And a and b are given as

$$a = \mu_{\underset{\sim}{e1}}(e) = \frac{e2 - e}{e2 - e1}, \qquad b = \mu_{\underset{\sim}{\Delta e1}}(\Delta e) = \frac{\Delta e2 - \Delta e}{\Delta e2 - \Delta e1}. \tag{20}$$



(a) Case of product-sum-gravity method and simplified fuzzy reasoning method
(b) Case of min-max-gravity method

Fig. 8. Inference result u from e and Δe in (17).

Therefore, the inference result (i.e., control action) u can be derived as follows by inserting (18) and (20) into (19):

$$u = (19)$$

$$= \alpha e + \beta \Delta e \tag{21}$$

which indicates the construction of PD controller by means of a simplified reasoning method (see Fig. 8(a)).

In the same way, we can realize PD controller under the product–sum-gravity method by extending the consequent part $u1$, $u2$, $u3$ and $u4$ of (17) to fuzzy sets $\underset{\sim}{u1}$ (around $u1$), $\underset{\sim}{u2}$, $\underset{\sim}{u3}$ and $\underset{\sim}{u4}$ of isosceles triangular type with the same width.

Note that min–max-gravity method cannot realize such PD controller since it does not generate a plane as shown in Fig. 8(b).

We shall next consider the case of *PID controllers* of (14).

Let ie be the integral value $\int e\,dt$ of an error e, and its minimum and maximal values be $ie1$ and $ie2$, respectively. Namely,

$$ie1 \leqslant ie \leqslant ie2. \tag{22}$$

Fuzzy sets $\underset{\sim}{ie1}$ and $\underset{\sim}{ie2}$ are given in Fig. 6(c).

Then the fuzzy control rules for PID controllers are given as

$$\begin{array}{lllll}
\text{Rule 1:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e1} \text{ and } \underset{\sim}{ie1} & \Rightarrow u1 \\
\text{Rule 2:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e1} \text{ and } \underset{\sim}{ie2} & \Rightarrow u2 \\
\text{Rule 3:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e2} \text{ and } \underset{\sim}{ie1} & \Rightarrow u3 \\
\text{Rule 4:} & \underset{\sim}{e1} \text{ and } \underset{\sim}{\Delta e2} \text{ and } \underset{\sim}{ie2} & \Rightarrow u4 \\
\text{Rule 5:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e1} \text{ and } \underset{\sim}{ie1} & \Rightarrow u5 \\
\text{Rule 6:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e1} \text{ and } \underset{\sim}{ie2} & \Rightarrow u6 \\
\text{Rule 7:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e2} \text{ and } \underset{\sim}{ie1} & \Rightarrow u7 \\
\text{Rule 8:} & \underset{\sim}{e2} \text{ and } \underset{\sim}{\Delta e2} \text{ and } \underset{\sim}{ie2} & \Rightarrow u8 \\
\text{Fact:} & e \text{ and } \Delta e \text{ and } ie & \\
\hline
\text{Cons:} & & u
\end{array} \tag{23}$$

where $u1$, $u2 \ldots, u8$ are real numbers such that

$$\begin{aligned} u1 &= \alpha e1 + \beta\,\Delta e1 + \gamma\, ie1 \\ u2 &= \alpha e1 + \beta\,\Delta e1 + \gamma\, ie2 \\ u3 &= \alpha e1 + \beta\,\Delta e2 + \gamma\, ie1 \\ u4 &= \alpha e1 + \beta\,\Delta e2 + \gamma\, ie2 \\ u5 &= \alpha e2 + \beta\,\Delta e1 + \gamma\, ie1 \\ u6 &= \alpha e2 + \beta\,\Delta e1 + \gamma\, ie2 \\ u7 &= \alpha e2 + \beta\,\Delta e2 + \gamma\, ie1 \\ u8 &= \alpha e2 + \beta\,\Delta e2 + \gamma\, ie2 \end{aligned} \tag{24}$$

which are the values of (14) at the points $(e1, \Delta e1, ie1), \ldots, (e2, \Delta e2, ie2)$.

Thus, the control action u for e, Δe and ie is given as

$$\begin{aligned} u &= \frac{\begin{array}{l} abcu1 + ab(1-c)u2 + a(1-b)cu3 + a(1-b)(1-c)u4 \\ + (1-a)bcu5 + (1-a)b(1-c)u6 + (1-a)(1-b)cu7 \\ + (1-a)(1-b)(1-c)u8 \end{array}}{\begin{array}{l} abc + ab(1-c) + a(1-b)c + a(1-b)(1-c) \\ + (1-a)bc + (1-a)b(1-c) + (1-a)(1-b)c \\ + (1-a)(1-b)(1-c) \end{array}} \\ &= abcu1 + ab(1-c)u2 + a(1-b)cu3 + a(1-b)(1-c)u4 \\ &\quad + (1-a)bcu5 + (1-a)b(1-c)u6 + (1-a)(1-b)cu7 \\ &\quad + (1-a)(1-b)(1-c)u8 \\ &= \alpha e + \beta\,\Delta e + \gamma \int e\,\mathrm{d}t, \end{aligned} \tag{25}$$

where

$$c = \mu_{\underset{\sim}{ie1}}(ie) = \frac{ie2 - ie}{ie2 - ie1}. \tag{26}$$

Therefore, it is shown that PID controller can be constructed by the simplified reasoning method. The same holds for product–sum-gravity method. Min–max-gravity, however, cannot realize PID controller.

In the above discussion, we have treated the case where intervals $[e1, e2]$, $[\Delta e1, \Delta e2]$, $[ie1, ie2]$ are divided into two fuzzy sets as shown in Fig. 6. We shall next discuss the general case where these intervals are divided into more than two fuzzy sets.

Let the interval of possible error e be partitioned into m fuzzy sets as shown in Fig. 9(a) so that the neighboring fuzzy sets can intersect at the height 0.5. In the same way, the intervals of change in error Δe and integral value ie are divided into n fuzzy sets and p fuzzy sets, respectively, as in Fig. 9(b) and (c).

Fuzzy control rules which realize PD controller by the simplified fuzzy reasoning method are given as

$$\begin{aligned} &\text{Rule } 1\text{:} \quad \underset{\sim}{e_1} \text{ and } \Delta\underset{\sim}{e_1} \Rightarrow u_1 \\ &\qquad \vdots \\ &\text{Rule } ij\text{:} \quad \underset{\sim}{e_i} \text{ and } \Delta\underset{\sim}{e_j} \Rightarrow u_{ij} \\ &\qquad \vdots \\ &\text{Rule } mn\text{:} \quad \underset{\sim}{e_m} \text{ and } \Delta\underset{\sim}{e_n} \Rightarrow u_{mn} \end{aligned} \tag{27}$$

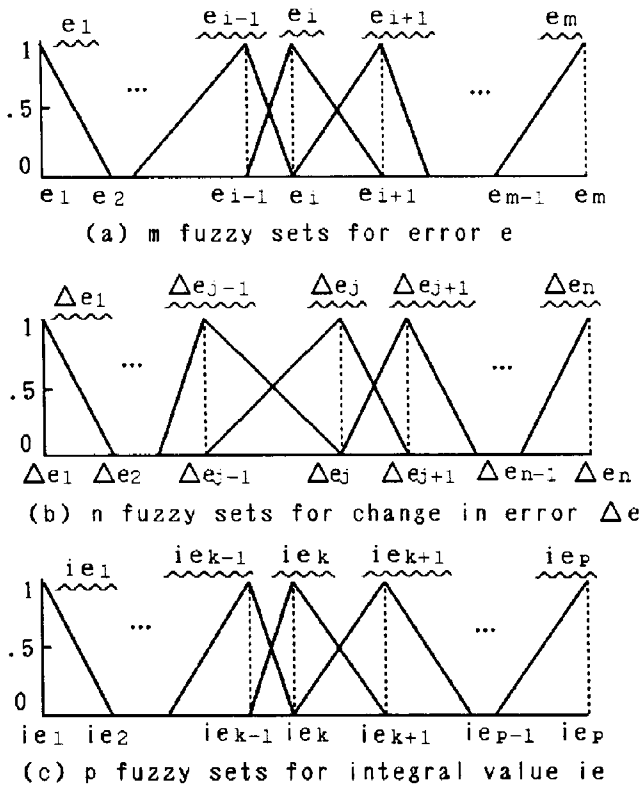


Fig. 9. Fuzzy sets for error e, change in error Δe and integral value ie ($=\int e\,dt$).

where the consequent part u_{ij} is given as

$$u_{ij} = \alpha e_i + \beta\, \Delta e_j \tag{28}$$

for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

In the same way, we can construct PID controller using fuzzy control rules such as

$$\text{Rule } ijk\text{:} \quad \underset{\sim}{e_i} \text{ and } \underset{\sim}{\Delta e_j} \text{ and } \underset{\sim}{ie_k} \Rightarrow u_{ijk} \tag{29}$$

with

$$u_{ijk} = \alpha e_i + \beta\, \Delta e_j + \gamma\, ie_k \tag{30}$$

for $i = 1, \ldots, m$, $j = 1, \ldots, n$ and $k = 1, \ldots, p$.

Similarly, PID controller can be realized by product–sum-gravity method by extending the consequent part u_{ijk} to fuzzy set $\underset{\sim}{u_{ijk}}$ with the same width. But it is shown that min–max-gravity method cannot realize PID controller.

4. Extrapolative reasoning by fuzzy reasoning methods

In the discussion of the realization of PD controller and PID controller, it is assumed that e, Δe and ie are in the intervals between the minimum value and the maximal value as in (16) and (22), and thus the grades a, b, c of (20) and (26) are in the unit interval $[0, 1]$. However, in the derivations of u in (21) and (25), we have not used the condition that a, b and c are in $[0, 1]$, which suggests that fuzzy sets, say, $\underset{\sim}{e1}$ and $\underset{\sim}{e2}$ should not necessarily be of triangular type with height 1, but may be characterized by such functions as in Fig. 10 with the range $(-\infty, \infty)$ rather than $[0, 1]$.

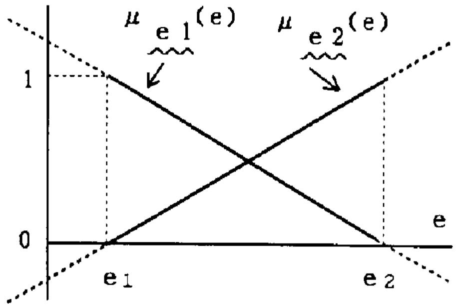
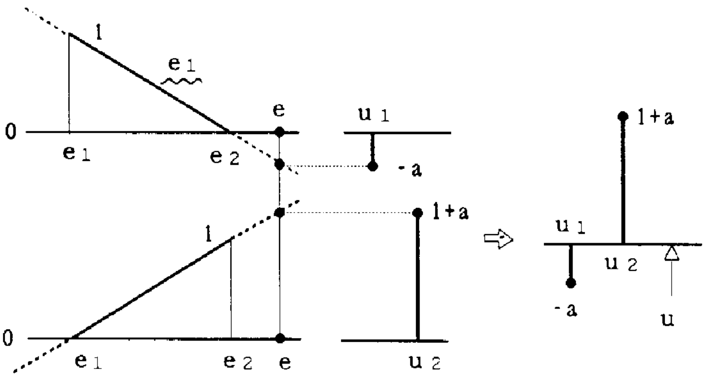
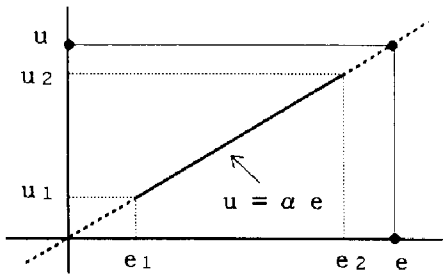


Fig. 10. Membership functions extended to $(-\infty, \infty)$.



(a) Illustration of extrapolative reasoning



(b) Result of extrapolation

Fig. 11. Extrapolative reasoning and extrapolation result.

As a simple example, we shall consider *P controller* ($u = \alpha e$) which is realized by the following two fuzzy rules:

$$\begin{aligned} &\text{Rule 1:} \quad \underset{\sim}{e1} \Rightarrow u1 \\ &\text{Rule 2:} \quad \underset{\sim}{e2} \Rightarrow u2 \end{aligned} \tag{31}$$

where $u1 = \alpha e1$ and $u2 = \alpha e2$. $e1$ and $e2$ are arbitrary values (thus, not necessarily minimal and maximal values) for e such as $e1 < e2$, and $\underset{\sim}{e1}$ and $\underset{\sim}{e2}$ are characterized by such functions as in Fig. 10 whose range is $(-\infty, \infty)$.

Then the control action u at e (say, e is outside $[e1, e2]$ as in Fig. 11) is given as

$$u = \frac{-au1 + (1+a)u2}{-a + (1+a)} = -au1 + (1+a)u2 = \alpha e, \tag{32}$$

where $-a = (20)$.

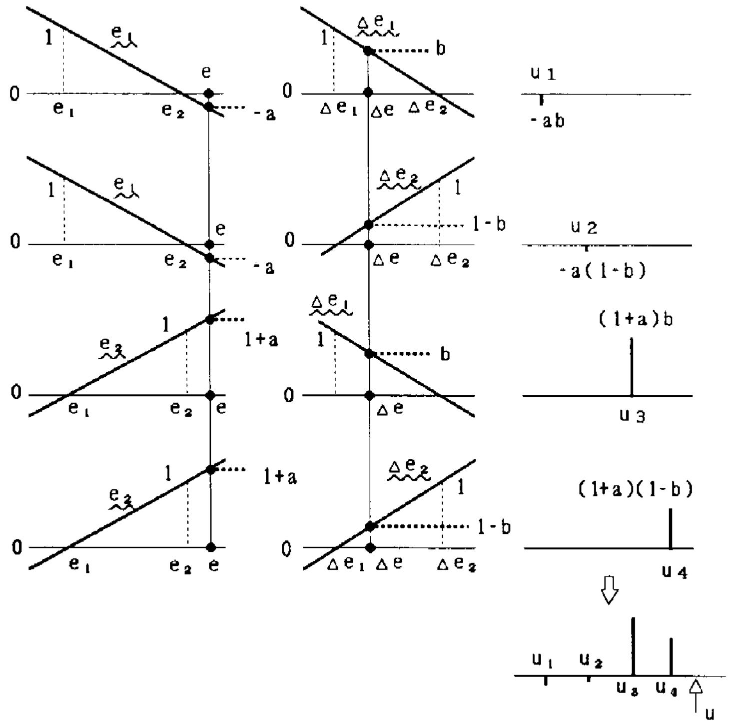


Fig. 12. Extrapolative reasoning of (17).

It is found from Fig. 11 that the inference result u is given by dividing externally $u1$ and $u2$ in the ratio $1 + a : a$. Namely, when e is outside $[e1, e2]$, u is given as the external division point of $u1$ and $u2$, which indicates the possibility of extrapolation. Obviously, when e is in $[e1, e2]$, u is in $[u1, u2]$.

PD controller is discussed in the same way. Fuzzy control rules for PD controller are given in (17), where $\Delta e1$ and $\Delta e2$ are also arbitrary values for Δe such as $\Delta e1 < \Delta e2$, and the range of membership functions of fuzzy sets $\underset{\sim}{\Delta e1}$ and $\underset{\sim}{\Delta e2}$ is also extended from $[0, 1]$ to $(-\infty, \infty)$. Thus, the control action u in the case of, say, $e \notin [e1, e2]$ and $\Delta e \in [\Delta e1, \Delta e2]$ is inferred as in Fig. 12 and is given as $u = \alpha e + \beta\, \Delta e$ of (15).

Similarly, we can discuss PID controllers by extending the ranges of membership functions of $\underset{\sim}{e1}$, $\underset{\sim}{e2}$, $\underset{\sim}{\Delta e1}$, $\underset{\sim}{\Delta e2}$, $\underset{\sim}{ie1}$ and $\underset{\sim}{ie2}$ from $[0, 1]$ to $(-\infty, \infty)$.

We can also discuss such an extrapolative reasoning in the case of fuzzy sets in Fig. 9 by extending the range of membership functions of the fuzzy sets just at both ends to $(-\infty, \infty)$.

5. Conclusion

We have shown that PID controllers can be realized by fuzzy control methods of "product–sum-gravity method" and "simplified fuzzy reasoning method". Therefore, PID controls are regarded as a special case of fuzzy controls. Min–max-gravity method, however, cannot realize PID controllers.

As was shown in (17) and (23), PD controllers are realized by 4 fuzzy control rules and PID controllers by 8 control rules. These fuzzy controllers, however, will not be used as PID controllers in the practical use because of the slow speed. But if the consequent parts $u1, u2, \ldots$ of the fuzzy control rules of, say, (23) are altered independently, we can construct a fuzzy controller which is a generalized version of PID controller.

Extrapolative reasoning is proposed and shown to be executed by the product–sum-gravity method and simplified fuzzy reasoning method by extending the range of membership functions of antecedent parts of fuzzy rules from $[0, 1]$ to $(-\infty, \infty)$, which suggests that, in general, the range of membership functions of fuzzy control rules are not necessarily in the unit interval [0, 1]. In fact, fuzzy control methods are proposed in [6] to realize emphatic and suppressive effects on fuzzy control rules in which the range of membership functions of "consequent" part of fuzzy control rules is extended to $(-\infty, \infty)$.

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