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FUZZY CONTROLS BY PRODUCT-SUM-GRAVITY METHOD DEALING WITH FUZZY RULES OF EMPHATIC AND SUPPRESSIVE TYPES

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This paper shows that emphatic effects on fuzzy inference results are observed under product-sum-gravity method by using fuzzy control rules whose consequent part is characterized by a membership function whose grades are greater than 1. Suppressive effects are also realized by employing fuzzy control rules whose consequent part is characterized by a negative-valued membership function.It is shown that good control results are obtained by using the fuzzy control rules of emphatic and suppressive types.

1. Introduction

This paper proposes new type of "product-sum-gravity method" in which a fuzzy set of consequent part of fuzzy control rule is characterized by a membership function whose range is $(-\infty, \infty)$ rather than $[0, 1]$. Emphatic effects can be realized by employing fuzzy control rules whose consequent part is characterized by a membership function whose grades are greater than 1. Suppressive effects are also realized by using fuzzy control rules whose consequent part is characterized by a negative-valued membership function.

It is shown that good control results are obtained by using the fuzzy control rules of emphatic and suppressive types since this method can subtly adjust control results by changing the height of consequent part of fuzzy control rules.

2. Product-Sum-Gravity Method

We shall consider the following multiple fuzzy reasoning form:

$$\begin{array}{ll} \text{Rule 1}: & A_1 \text{ and } B_1 \;\Rightarrow C_1 \\ \text{Rule 2}: & A_2 \text{ and } B_2 \;\Rightarrow C_2 \\ & \cdots\cdots\cdots\cdots \\ \text{Rule n}: & A_n \text{ and } B_n \;\Rightarrow C_n \\ \text{Fact} \;\; : & x_0 \text{ and } y_0 \\ \hline \text{Cons} \;\; : & \qquad\qquad\quad C' \end{array} \tag{1}$$

where A_i is a fuzzy set in X; B_i in Y; and C_i in Z and $x_0 \in X$, $y_0 \in Y$.

We shall begin with a fuzzy reasoning method called *product-sum-gravity method* [1–3] (see Fig.1).

Each inference result C_i' which is inferred from the fact "x_0 and y_0" and the fuzzy rule "A_i and $B_i \Rightarrow C_i$" is given as

$$\mu_{C_i'}(z) = \mu_{A_i}(x_0) \cdot \mu_{B_i}(y_0) \cdot \mu_{C_i}(z) \tag{2}$$

where $\cdot$ stands for algebraic product. The final consequence C' of (1) is aggregated by taking the sum (addition $+$) of $C_1', C_2', \cdots, C_n'$ obtained above. Namely,

$$\mu_{C'}(z) = \mu_{C_1'}(z) + \cdots + \mu_{C_n'}(z) \tag{3}$$

The representative point z_0 for the resulting fuzzy set C' is given as the center of gravity of C':

$$z_0 = \frac{\int z \cdot \mu_{C'}(z)}{\int \mu_{C'}(z)} \tag{4}$$

Note that it is possible to define *min-max-gravity method* known as Mamdani's fuzzy control method [4] by replacing product with min in (2) and sum with max in (3).

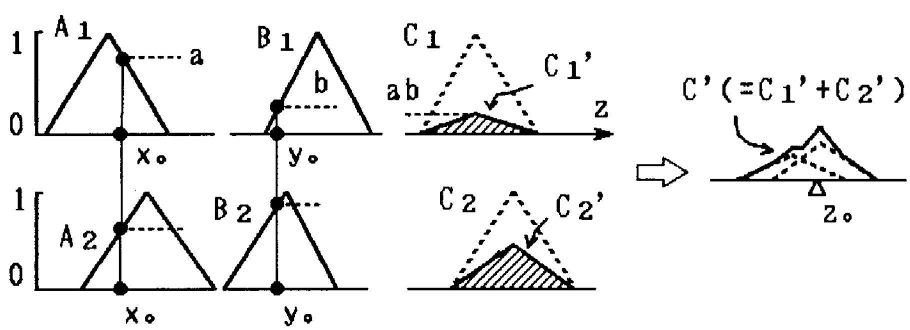


Fig.1 Product-sum-gravity method

3. Fuzzy Control Rules of Emphatic and Suppressive Types

In the product-sum-gravity method, the consequence C' is obtained by summing up the inference results C_i' as shown in (3). The use of sum operation rather than max operation enables us to define fuzzy control rules of emphatic and suppressive types.

3.1. *Fuzzy control rules of emphatic type*

Usually, an identical fuzzy control rule is not used two or many times at a time in the execution of fuzzy controls. However, an identical fuzzy control rule can be used twice or more in the product-sum-gravity method and thus emphatic effects are obtained on fuzzy inference results.

For example, suppose that a fuzzy control rule A and $B \Rightarrow C$ is used twice, that is,

$$\begin{array}{l} \text{Rule 1}: \ A \text{ and } B \Rightarrow C \\ \text{Rule 2}: \ A \text{ and } B \Rightarrow C \end{array} \tag{5}$$

are used simultaneously, then the inference result obtained is double, i.e., $2ab$ in height as shown in Fig.2 because of the use of addition operation in the aggregation of the same two inference results. Hence, using the identical fuzzy rule twice, we can have an inference result at double height and thus double emphatic effect is given on the inference result. Therefore, the role of the fuzzy control rule is enhanced every time it is used twice or more at the same time.

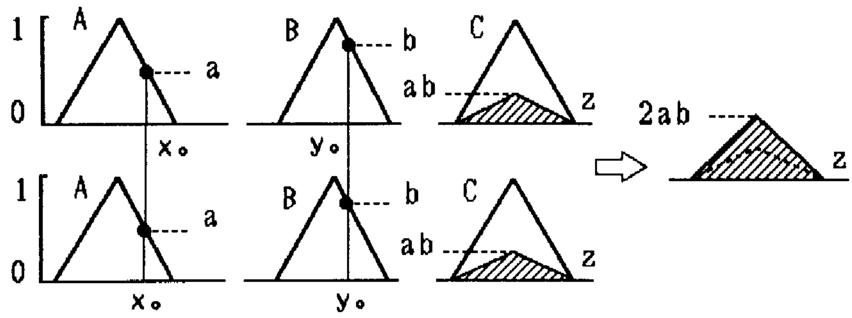


Fig.2 Emphatic effect by using two same rules

In general, to obtain emphatic effect of w times, it is suggested to use a fuzzy control rule A and $B \Rightarrow C$ whose consequent part C consists of a fuzzy set whose height is w rather than 1.

w is assumed to be a non-negative real number ($w \geq 0$) and thus w is not necessarily in $[0, 1]$. $w > 1$ means that the corresponding rule A and $B \Rightarrow C$ is "emphasized," and $0 \leq w < 1$ means that the rule is "restrained."

Fig.3 shows a fuzzy rule whose consequent part is w in height. Clearly, the same inference result as Fig.2 is obtained when $w = 2$. The reasoning process for fuzzy rules of emphatic type is shown in Fig.4.

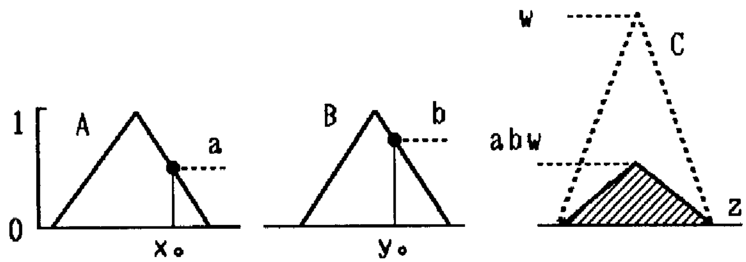


Fig.3 Emphatic effect of w times

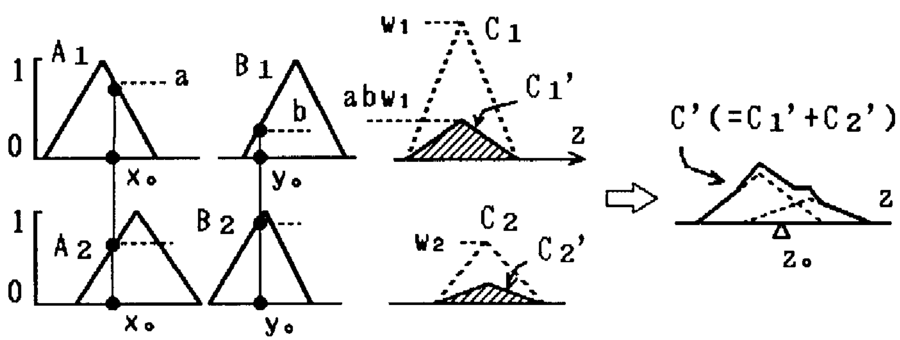


Fig.4 Product-sum-gravity method for fuzzy rules of emphatic type

It should be noted that such emphatic effects are not vealized in the case of min-max-gravity method by Mamdani because of the use of "max" operation in the aggregation of inference results.

3.2. *Fuzzy control rules of suppressive type*

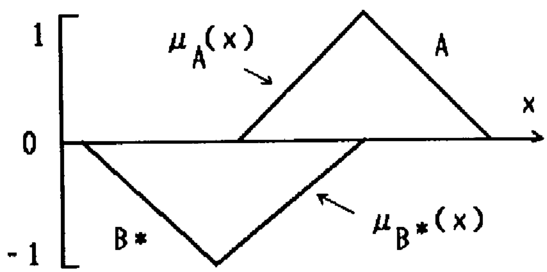
At first, we shall consider an ordinary fuzzy set A and a special fuzzy set $B*$ which is characterized by a "negative" membership function μ_{B*}. Clearly, we have

$$-1 \leq \mu_{B*}(x) \leq 0 \leq \mu_A(x) \leq 1 \tag{6}$$

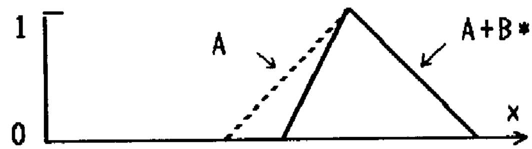
We shall define the sum of A and $B*$ as

$$\mu_{A+B*}(x) = [\mu_A(x) + \mu_{B*}(x)] \vee 0 \tag{7}$$

For example, it is found from the result of $A + B*$ in Fig.5(b) that the left part of fuzzy set A is cut down by the negative fuzzy set $B*$, which indicates that the left part of A is suppressed by $B*$.



(a) Negative fuzzy set $B*$



(b) Sum $A + B*$ of A and $B*$

Fig.5 Sum $A + B*$ of fuzzy set A and negative fuzzy set $B*$

We shall next show that a suppressive effect is obtained on the inference results when fuzzy sets of the consequent part of fuzzy control rules are characterized by negative-valued membership functions.

As a simple example, we shall consider the following three fuzzy control rules such that the consequent part C_1* of the second fuzzy rule is defined by a negative-valued membership function of trapezoidal type as in Fig.6.

$$\begin{aligned} &\text{Rule 1: } A_1 \text{ and } B_1 \Rightarrow C_1 \\ &\text{Rule 2: } A_1 \text{ and } B_1 \Rightarrow C_1* \\ &\text{Rule 3: } A_2 \text{ and } B_2 \Rightarrow C_2 \end{aligned} \tag{8}$$

The inference result C_1' by Rule 1 is given as $\mu_{C_1'}(z) = ab\mu_{C_1}(z)$ and is of a triangular type at height ab (See Fig.7).

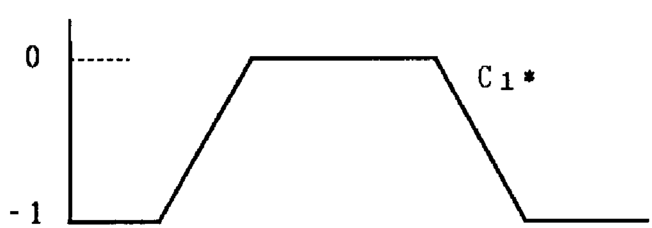


Fig.6 Negative fuzzy set C_1*

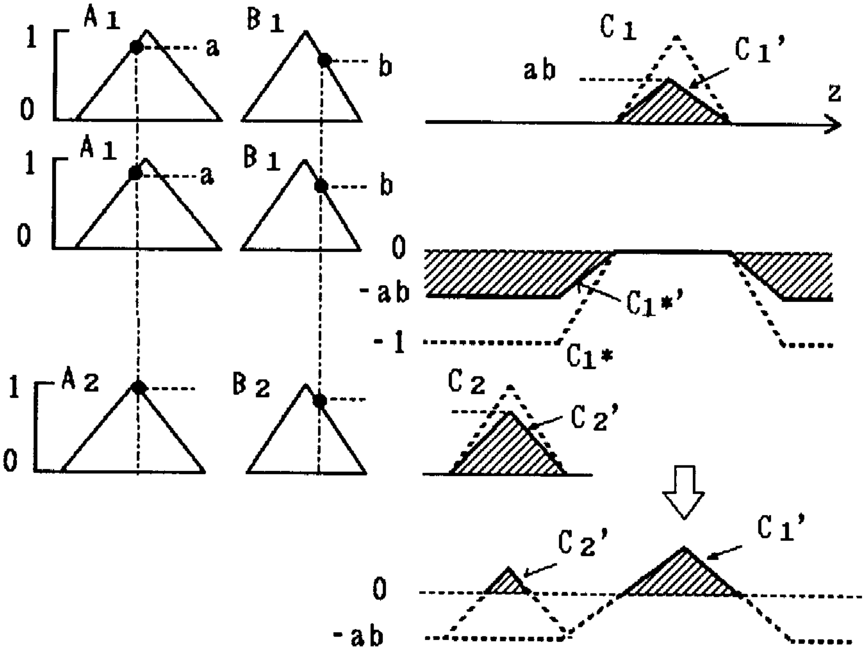


Fig.7 Reasoning process of fuzzy rules with negative consequent part and its aggregation result

On the other hand, the inference result C_1*' from Rule 2 is also given as

$$\mu_{C_1*'}(z) = ab\mu_{C_1*}(z) \tag{9}$$

But the result C_1*' is of a trapezoidal type of depth $-ab$ as shown in Fig.7 because $\mu_{C_1*}(z)$ is negative. Therefore, it is found from the aggregated result in Fig.7 that the inference result C_2' by Rule 3 is suppressed by the negative inference result C_1*' and thus the resulting C_2' becomes smaller than the original C_2'. Therefore, since C_2' becomes small compared with C_1', the influence of C_1' becomes strong relatively.

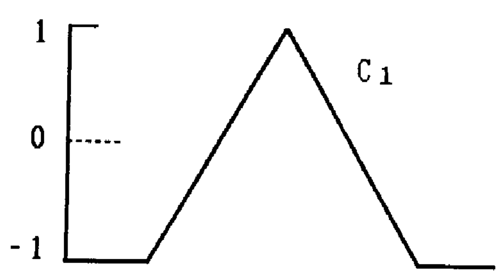


Fig.8 Consequent part of mixed types of emphasis and suppression

It is noted that in the case of defining a fuzzy control rule of suppressive type, the membership function is assumed to be negative. But it is possible to use a mixed type of membership function whose values are positive or negative as shown in Fig.8. More generally, we can introduce a membership function whose range $[-1, 1]$ is extended to $(-\infty, \infty)$ in order to obtain more general emphatic and suppressive effects.

Fig.9 shows the reasoning process equivalent to Fig.7 where consequent part consists of a mixed type of fuzzy set as in Fig.8.

Next we shall indicate the control results using fuzzy control rules of emphatic and suppressive types.

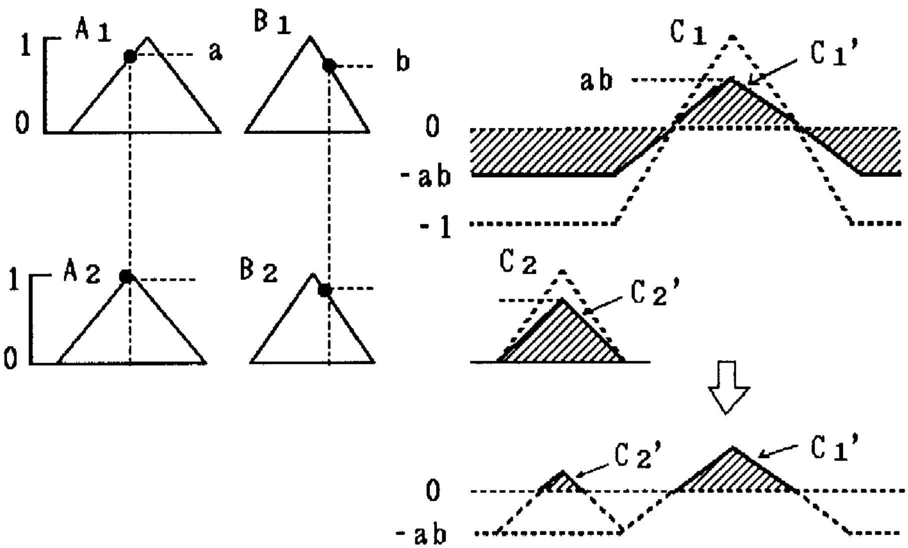


Fig.9 Reasoning process equivalent to Fig.7

4. Fuzzy Control Results

In this section we shall show the control results when fuzzy control rules of emphatic and suppressive types are used in the product-sum-gravity method.

Now we shall consider a plant model $G(s) = e^{-2s}/(1 + 20s)$ with first order delay. Fuzzy control rules for the plant model are shown in the table in Fig.11 and interpretted as

$$\begin{array}{rl} \text{R1}: & e \text{ is } NB \text{ and } \Delta e \text{ is } NB \Rightarrow \Delta u \text{ is } PB \\ \text{R2}: & e \text{ is } NB \text{ and } \Delta e \text{ is } NS \Rightarrow \Delta u \text{ is } PB \\ & \cdots\cdots\cdots\cdots\cdots\cdots \\ \text{R25}: & e \text{ is } PB \text{ and } \Delta e \text{ is } PB \Rightarrow \Delta u \text{ is } NB \end{array} \tag{10}$$

where e is error, Δe is change in error and Δu is change in action. NB, NS, $\cdots$, PB are fuzzy sets in Fig.10.

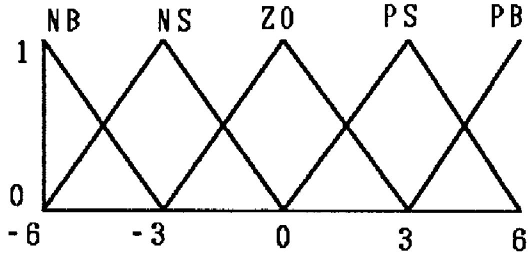


Fig.10 Fuzzy sets

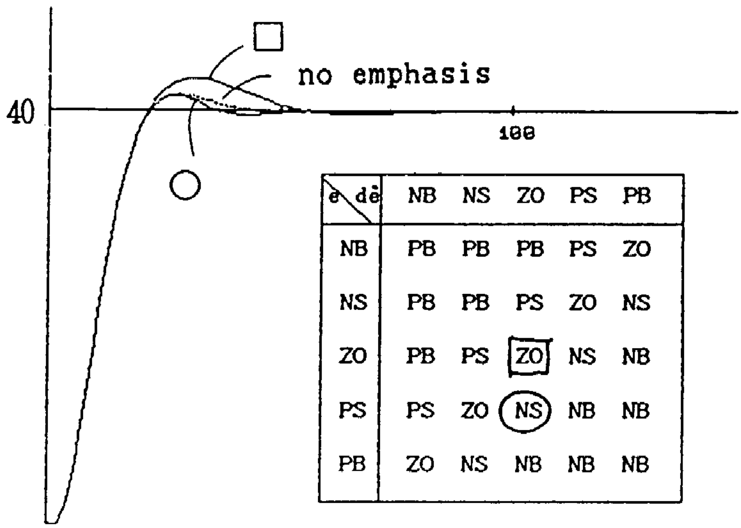
Now we shall begin with the case of using fuzzy control rules of emphatic types. The conditions are as follows:

$$\begin{aligned} &\text{Time constant} = 20; \quad \text{Dead time} = 2 \\ &\text{Scale factor of } \Delta e = 1.2 \\ &\text{Scale factor of } \Delta u = 2.5 \end{aligned} \tag{11}$$

In order to emphasize that at the portion just over the set point 40, i.e. "e is PS and Δe is ZO," the change in error should be decreased a little "Δu is NS," the fuzzy control rule

$$e \text{ is } PS \text{ and } \Delta e \text{ is } ZO \Rightarrow \Delta u \text{ is } NS \tag{12}$$

which is indicated by ○ in table of Fig.11, is used twice. As a result, the overshoot is pressed down a little. It is noted that the dotted line shows the control result in the case of no emphasis.



e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	PS	ZO
NS	PB	PB	PS	ZO	NS
ZO	PB	PS	[ZO]	NS	NB
PS	PS	ZO	(NS)	NB	NB
PB	ZO	NS	NB	NB	NB

Fig.11 Control results in case of emphasis

The symbol □ shows that the rule

$$e \text{ is } ZO \text{ and } \Delta e \text{ is } ZO \Rightarrow \Delta u \text{ is } ZO \tag{13}$$

is used twice so that the emphasis effect near the set point is aimed. But the control result becomes bad since the change in error Δu is emphasized to be zero near the set point and thus the response turns to be dull.

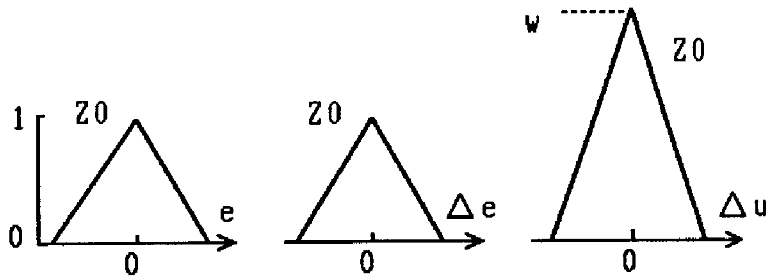


Fig.12 Fuzzy rule ZO and $ZO \Rightarrow ZO$

In connection with this, let the height of ZO in "Δu is ZO" of (13) be w as shown in Fig.12, which is indicated as ZO^w at the center of control table of Fig.13. Clearly, the height $w = 2$ corresponds to the case of using the fuzzy control rule (13) twice.

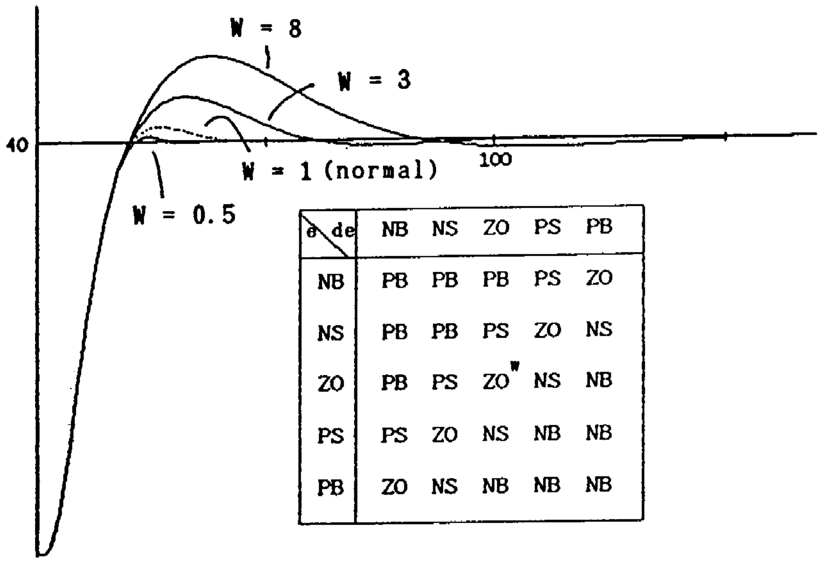
In order to observe the emphatic effect near the setting point 40, we shall show the control results in Fig.13 obtained by changing the height w of Fig.12.

At first, let w be large as is $w = 3$ and 8 to realize the emphatic effect at the setting point, which corresponds to make $\Delta u = 0$ forcibly (namely, to make the control action u unchangeable), then the control result becomes bad as shown in Fig.13(a) since the change in error Δu is emphasized to be zero near the set point and thus the response turns to be dull.

It is noted that dotted line in Fig.13 shows the control result at $w = 1$, which corresponds to the case of no emphasis.

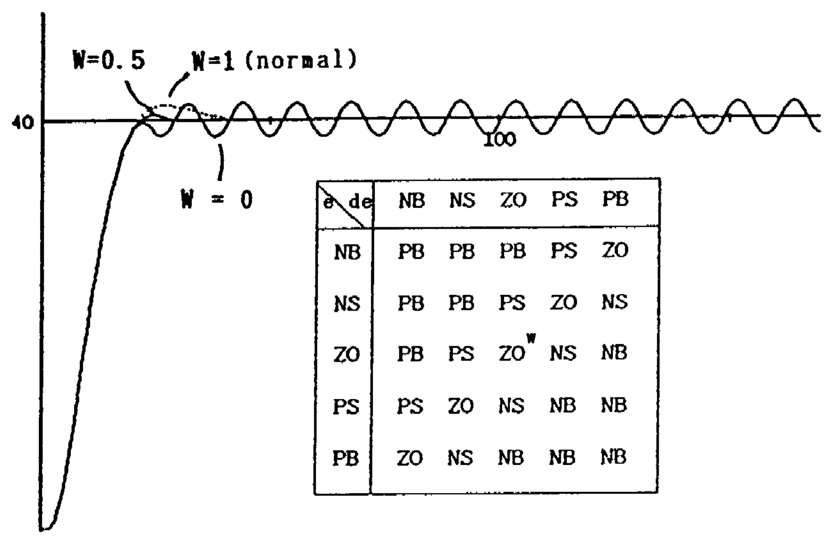
On the contrary, when w is small, the restrained effect works and the best control result is found at $w = 0.5$ as in Fig.13(a) and (b). However, at $w = 0$, the control result becomes periodical. $w = 0$ means no existence of fuzzy control rule (13), so an actual Δu is decided by the surrounding control rules and thus the periodical control result occurs.

Fig.14 shows the inference results Δu at e and Δe in the cases of $w = 1$, 0.5 and 0 for the control table in Fig.13. Fig.14(a) shows the case of $w = 1$ which corresponds to the ordinary case of no emphasis. Fig.14(b) is the case of $w = 0.5$ which gives the best control result and the vicinity of center of the figure is not a plane but changes slightly, which causes good control result. Fig.14(c) is the case of $w = 0$ which indicates no existence of fuzzy rule (13).



e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	PS	ZO
NS	PB	PB	PS	ZO	NS
ZO	PB	PS	ZO^{W}	NS	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

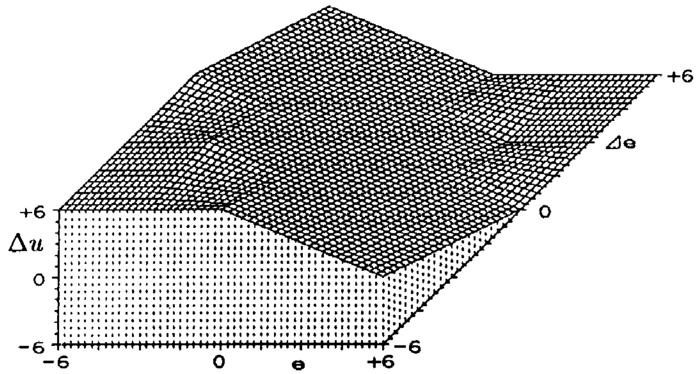
(a) Case of large w (emphatic case)



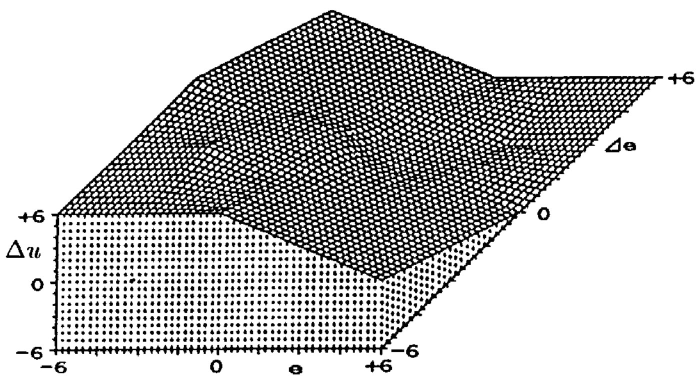
e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	PS	ZO
NS	PB	PB	PS	ZO	NS
ZO	PB	PS	ZO^{W}	NS	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

(b) Case of small w (restrained case)

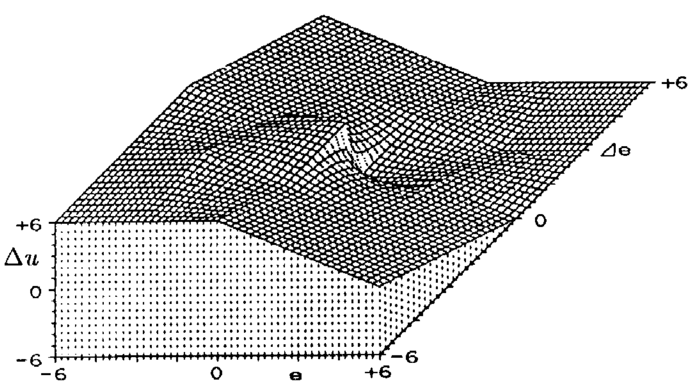
Fig.13 Control results when w of Fig.12 is changed



(a) Case of $w = 1$

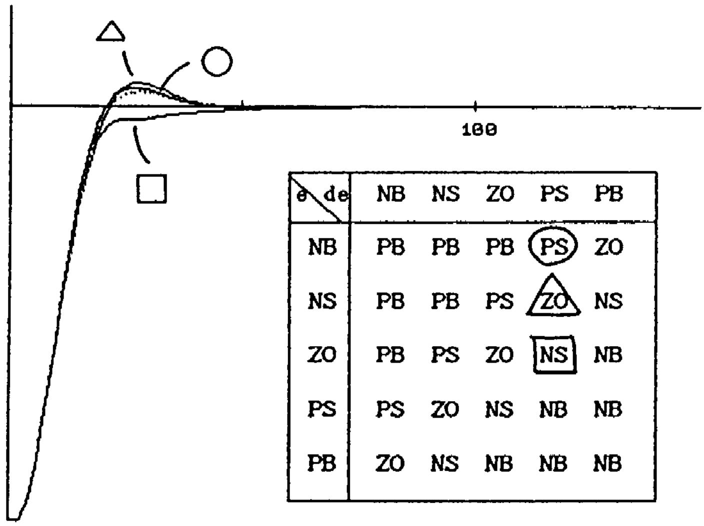


(b) Case of $w = 0.5$



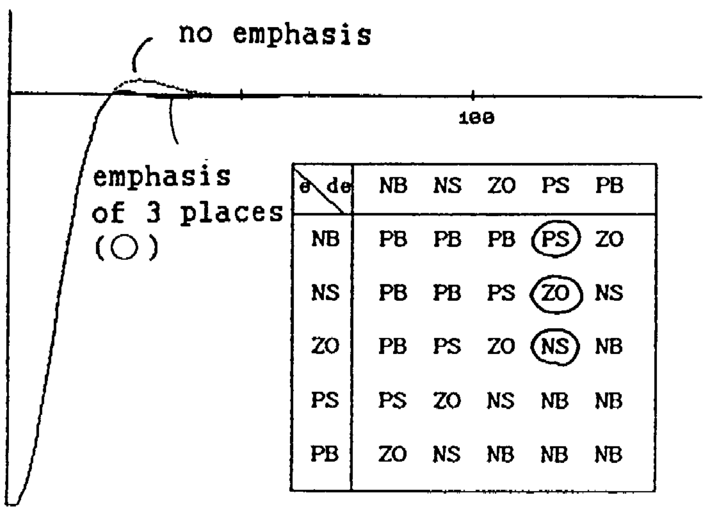
(c) Case of $w = 0$

Fig.14 Illustrations of e, $\Delta e \rightarrow \Delta u$ at $w = 1$, 0.5 and 0 for the control table in Fig.13



e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	(PS) ○	ZO
NS	PB	PB	PS	(ZO) △	NS
ZO	PB	PS	ZO	(NS) □	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

(a) Cases of emphasizing each control rule



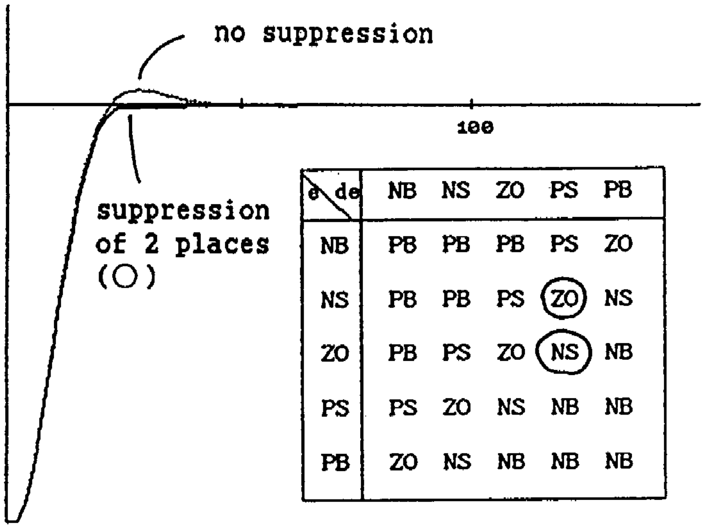
e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	(PS) ○	ZO
NS	PB	PB	PS	(ZO) ○	NS
ZO	PB	PS	ZO	(NS) ○	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

(b) Cases of emphasizing 3 control rules(○)

Fig.15 Contrl results by emphasizing each control rule (a) and 3 control rules simultaneously (b)

Fig.15(a) shows three control results when the individual fuzzy control rules ○, △, □ are emphasized independently. These control results are all not good. However, when the three control rules are emphasized simultaneously, the control result becomes quite good as shown in Fig.15(b) since the effects of the overshoot and the undershoot are counteracted.

We shall next show the suppressive effect by the fuzzy control rules whose conclusion part is characterized by a negative-valued membership function which is of trapezoidal type as shown in Fig.6.



e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	PS	ZO
NS	PB	PB	PS	(ZO)	NS
ZO	PB	PS	ZO	(NS)	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

Fig.16 Case of suppressing 2 places(○)

Fig.16 indicates the control result when two control rules ○ of suppressive type are used at the same time, which shows the improvement of control result. In this case, we use such control rules as

$$e \text{ is } NS \text{ and } \Delta e \text{ is } PS \Rightarrow \Delta u \text{ is } ZO$$
$$e \text{ is } NS \text{ and } \Delta e \text{ is } PS \Rightarrow \Delta u \text{ is } ZO*$$
$$e \text{ is } ZO \text{ and } \Delta e \text{ is } PS \Rightarrow \Delta u \text{ is } NS$$
$$e \text{ is } ZO \text{ and } \Delta e \text{ is } PS \Rightarrow \Delta u \text{ is } NS*$$

where $ZO*$ and $NS*$ are characterized by negative membership functions which are of negative trapezoidal type as shown in Fig.17.

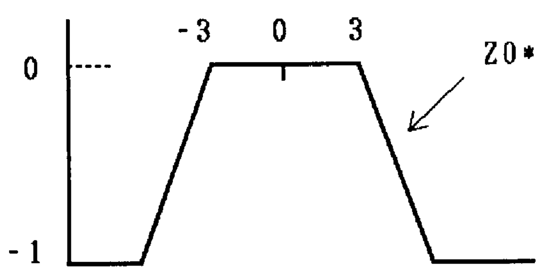
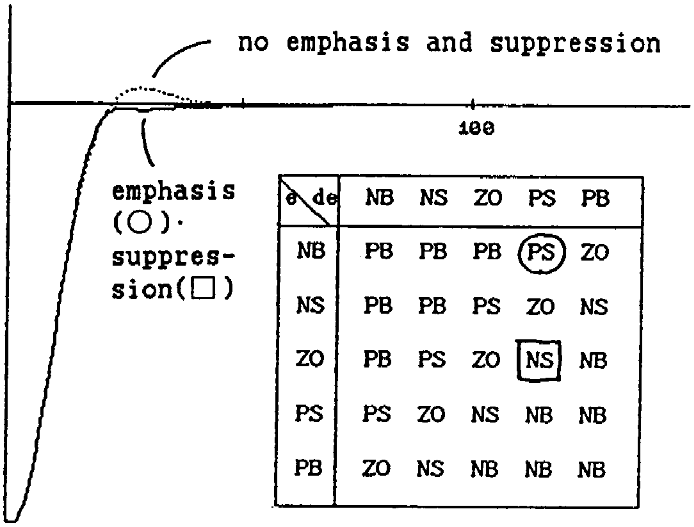


Fig.17 Negative fuzzy set $ZO*$



e \ de	NB	NS	ZO	PS	PB
NB	PB	PB	PB	(PS)	ZO
NS	PB	PB	PS	ZO	NS
ZO	PB	PS	ZO	[NS]	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

Fig.18 Case of simultaneous emphasis and suppression

Fig.18 shows the control result when the control rule ○ is emphasized and the control rule □ is suppressed at the same time.

5. Conclusion

This paper shows that better control results are obtained when fuzzy control rules of emphatic and suppressive types are used since the method enables us to adjust subtly the fuzzy control rules.

In the case of defining fuzzy control rules of emphatic and suppressive types, the range of membership function of consequent part of the fuzzy control rule is assumed to be $(-\infty, \infty)$ rather than $[0, 1]$. This suggests that we can define more general fuzzy control rules such that the antecedent part as well as the consequent part is characterized by a membership function whose range is not necessarily in the unit interval $[0, 1]$. In fact, it is shown in [3,5] that extrapolative reasoning is executed under product-sum-gravity method by extending the range of membership functions of antecedent parts of fuzzy control rules from $[0, 1]$ to $(-\infty, \infty)$.

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