Kacprzyk/Yager (Ed.): Management decision support systems using fuzzy sets and possibility theory
ISR 83 © Verlag TÜV Rheinland GmbH, Köln

FUZZY INFERENCE WITH "IF ... THEN ... ELSE ..." UNDER NEW COMPOSITIONAL RULES OF INFERENCE

Masaharu MIZUMOTO

Department of Management Engineering
Osaka Electro-Communication University
Neyagawa, Osaka 572, Japan

This paper shows that most of fuzzy inference methods by Zadeh and Mizumoto with a fuzzy conditional proposition "If x is A then y is B else y is C" can infer reasonable consequences when new compositions called "max-$\odot$ composition" and "max-$\blacktriangle$ Composition," where $\odot$ is bounded-product and $\blacktriangle$ is drastic product, are used in the compositional rule of inference, though reasonable consequences cannot always be obtained when using the max-min composition which is used usually in the compositional rule of inference.

Keywords: Fuzzy conditional inference, Compositional rule of inference, Bounded-product, Drastic product, Max-min Composition, Max-$\odot$ composition, Max-$\blacktriangle$ composition

1. INTRODUCTION

In much of human reasoning, the form of reasoning is approximate rather than exact as in the statement:

If a demand is large then a price will be high.
The demand of autos is highly large.
___
The price of autos will be very high.

Zadeh (1975), Mamdani (1977), and Mizumoto et al. (1979, 1980, 1982a) suggested methods for such reasoning in which the premise involves a fuzzy conditional proposition "If x is A then y is B," where A and B are fuzzy concepts.

As a generalization of such a fuzzy conditional inference, Zadeh (1975) also proposed a fuzzy conditional inference of the form:

Prem 1: If x is A then y is B else y is C.
Prem 2: x is A'.
___
Cons: y is D.

For this form of inference, he proposed methods for obtaining the consequence (Cons) from two premises (Prem 1 and Prem 2).

This paper shows that, although reasonable consequences are not obtained by Zadeh's methods and other alternative methods (Mizumoto, 1982b) when using the max-min composition used usually in the compositional rule of inference, their methods can get reasonable consequence if new compositions called "max-$\odot$ composition" and "max-$\blacktriangle$ composition" are used in the compositional rule of inference.

2. FUZZY CONDITIONAL INFERENCE WITH "IF ... THEN ... ELSE ..."

We shall discuss the following form of inference in which a fuzzy conditional proposition "If ... then ... else ..." is contained.

$$\begin{array}{ll} \text{Prem 1:} & \text{If } x \text{ is } A \text{ then } y \text{ is } B \text{ else } y \text{ is } C. \\ \text{Prem 2:} & x \text{ is } A'. \\ \hline \text{Cons:} & y \text{ is } D. \end{array} \qquad (1)$$

where x and y are the names of objects, and A, A', B, C and D are fuzzy concepts which are represented by fuzzy sets in universes of discourse U, U, V, V and V, respectively.

An example of such a form of inference is the following:

> If a demand is <u>large</u> then a price will be <u>high</u> else a price will be <u>fairly</u> <u>low</u>.
>
> The demand of autos is <u>fairly</u> <u>large</u>.
>
> ---
>
> The price of autos will be <u>more or less</u> <u>high</u>.

The prem 1 of the form "If x is A then y is B else y is C" in (1) may present a certain relationship between A and B,C. From this point of view, Zadeh (1975) gave two translation rules "Maxmin Rule" and "Arithmetic Rule" for translating the fuzzy conditional proposition "If x is A then y is B else y is C" into a fuzzy relation in U x V.

Let A, B and C be fuzzy sets in U, V and V, respectively, which are written as

$$A = \int_U \mu_A(u)/u; \qquad B = \int_V \mu_B(v)/v; \qquad C = \int_V \mu_C(v)/v \qquad (2)$$

then we have:

(i) <u>Maximin Rule Rm'</u>:

$$\begin{aligned} Rm' &= (A \times B) \cup (\neg A \times C) \\ &= \int_{U\times V} (\mu_A(u) \wedge \mu_B(v)) \vee ((1-\mu_A(u)) \wedge \mu_C(v)) \;/\; (u,v) \end{aligned} \qquad (3)$$

(ii) <u>Arithmetic Rule Ra'</u>:

$$\begin{aligned} Ra' &= (\neg A \times V \oplus U \times B) \cap (A \times V \oplus U \times C) \\ &= \int_{U\times V} 1 \wedge (1-\mu_A(u)+\mu_B(v)) \wedge (\mu_A(u)+\mu_C(v)) \;/\; (u,v) \end{aligned} \qquad (4)$$

where x, ∪, ∩, ¬ and ⊕ denote cartesian product, union, intersection, complement and bounded-sum for fuzzy sets, respectively.

For the proposition "If x is A then y is B else y is C", it is also possible to define translation rules Rb' and Rgg' which are based on the implication rules of binary logic and Godelian logic, respectively.

(iii) <u>Fuzzified Binary Rule Rb'</u>:

$$\begin{aligned} Rb' &= (\neg A \times V \cup U \times B) \cap (A \times V \cup U \times C) \\ &= \int_{U\times V} ((1-\mu_A(u)) \vee \mu_B(v)) \wedge (\mu_A(u) \vee \mu_C(v)) \;/\; (u,v) \end{aligned} \qquad (5)$$

(iv) Rule Rgg'

$$Rgg' = (A \times V \underset{g}{\Rightarrow} U \times B) \cap (7A \times V \underset{g}{\Rightarrow} U \times C)$$
$$= \int_{U\times V} (\mu_A(u) \underset{g}{\rightarrow} \mu_B(v)) \wedge (1-\mu_A(u) \underset{g}{\rightarrow} \mu_C(v)) \; / \; (u,v) \qquad (6)$$

where

$$\mu_A(u) \underset{g}{\rightarrow} \mu_B(v) = \begin{cases} 1 & \ldots \quad \mu_A(u) \le \mu_B(v) \\ \mu_B(v) & \ldots \quad \mu_A(u) > \mu_B(v) \end{cases}$$

Note that the implication $a \underset{g}{\rightarrow} b$ is based on the implication rule of G_{aleph} logic system by Goedel (Rescher, 1969).

The consequence D in Cons of (1) can be deduced from Prem 1 and Prem 2 by using the "max-min composition" o of the fuzzy set A' in U and the fuzzy relation in U x V given above. Thus, we can have the consequence D for each translation rule (3)-(6) by the following. For example, the consequence by the method Rm' is defined by

$$Dm = A' \circ Rm' = A' \circ [(A \times B) \cup (7A \times C)] \qquad (7)$$

and the membership function μ_{Dm} of Dm is given by

$$\mu_{Dm}(v) = \bigvee_u \{\mu_{A'}(u) \wedge [(\mu_A(u) \wedge \mu_B(v)) \vee ((1-\mu_A(u)) \wedge \mu_C(v))]\} \qquad (8)$$

In the same way,

$$Da = A' \circ Ra' = A' \circ [(7A \times V \oplus U \times B) \cap (A \times V \oplus U \times C)] \qquad (9)$$

$$Db = A' \circ Rb' = A' \circ [(7A \times V \cup U \times B) \cap (A \times V \cup U \times C)] \qquad (10)$$

$$Dgg = A' \circ Rgg' = A' \circ [(A \times V \underset{g}{\Rightarrow} U \times B) \cap (7A \times V \underset{g}{\Rightarrow} U \times C)] \qquad (11)$$

Next we shall define new compositions called "max-Θ composition" □ and "max-▲ composition" ▲ which will be used in the compositional rule of inference. The operations of "bounded-product" Θ and "drastic product" ▲ which are used to define these compositions are given as follows: For any $x, y \in [0, 1]$,

Bounded-Product: $$x \odot y = 0 \vee (x + y - 1) \qquad (12)$$

Drastic Product: $$x \blacktriangle y = \begin{cases} x & \ldots \quad y = 1 \\ y & \ldots \quad x = 1 \\ 0 & \ldots \quad x,y < 1 \end{cases} \qquad (13)$$

Using these operations, we can easily define new compositions of max-Θ composition "□" and max-▲ composition "▲" of a fuzzy set A in U and a fuzzy relation R in UxV.

Max-Θ Composition "□":

$$A \,\square\, R = \mu_{A\square R}(v) = \bigvee_u \{\mu_A(u) \odot \mu_R(u,v)\} \qquad (14)$$

Max-▲ Composition "▲":

$$A \blacktriangle R = \mu_{A\blacktriangle R}(v) = \bigvee_u \{\mu_A(u) \blacktriangle \mu_R(u,v)\} \qquad (15)$$

~~From the definitions of these compositions we can have the following properti~~es which will be useful in the discussion of the fuzzy conditional inference. The more detailed properties are found in Mizumoto (1981b,c, 198[illegible], 1983a).

Let A, A_1 and A_2 be fuzzy sets in U, and R, R_1 and R_2 be fuzzy relations in $U \times V$, and let $* \in \{\circ, \square, \blacktriangle\}$, then we obtain

$$A * (R_1 \cup R_2) = (A * R_1) \cup (A * R_2) \tag{16}$$

$$(A_1 \cup A_2) * R = (A_1 * R) \cup (A_2 * R) \tag{17}$$

$$A * (R_1 \cap R_2) \subseteq (A * R_1) \cap (A * R_2) \tag{18}$$

$$(A_1 \cap A_2) * R \subseteq (A_1 * R) \cap (A_2 * R) \tag{19}$$

From the definitions of these new compositions, we can deduce the consequence D of (1) by taking the composition of the fuzzy set A' and the fuzzy relation given in (3)-(6). For example, in the case of max-$\odot$ composition "$\square$", we have for the method Rm' of (3)

$$D_m = A' \square R_m' = A' \square [(A \times B) \cup (\neg A \times C)] \tag{20}$$

The membership function of Dm is given by

$$\mu_{Dm}(v) = \bigvee_u \{\mu_{A'}(u) \odot [(\mu_A(u) \wedge \mu_B(v)) \vee ((1-\mu_A(u)) \wedge \mu_C(v))]\} \tag{21}$$

In the same way, we have

$$D_a = A' \square [(\neg A \times V \oplus U \times B) \cap (A \times V \oplus U \times C)]$$

$$D_b = A' \square [(\neg A \times V \cup U \times B) \cap (A \times V \cup U \times C)]$$

$$D_{gg} = A' \square [(A \times V \underset{g}{\Rightarrow} U \times B) \cap (\neg A \times V \underset{g}{\Rightarrow} U \times C)]$$

Similarly, for the case of max-$\wedge$ composition "$\blacktriangle$",

$$D_m = A' \blacktriangle R_m' = A' \blacktriangle [(A \times B) \cup (\neg A \times C)] \tag{22}$$

$$\mu_{Dm}(v) = \bigvee_u \{\mu_{A'}(u) \wedge [(\mu_A(u) \wedge \mu_B(v)) \vee ((1-\mu_A(u)) \wedge \mu_C(v))]\} \tag{23}$$

$$D_a = A' \blacktriangle [(\neg A \times V \oplus U \times B) \cap (A \times V \oplus U \times C)]$$

$$D_b = A' \blacktriangle [(\neg A \times V \cup U \times B) \cap (A \times V \cup U \times C)]$$

$$D_{gg} = A' \blacktriangle [(A \times V \underset{g}{\Rightarrow} U \times B) \cap (\neg A \times V \underset{g}{\Rightarrow} U \times C)]$$

3. COMPARISON OF FUZZY INFERENCE METHODS UNDER NEW COMPOSITIONS

In this section we shall make comparisons of the fuzzy inference methods (3)-(6) under new compositions as well as max-min composition. We shall discuss only the case of A' = A in (1) for simplicity because of limitations of space. The way of obtaining the consequence D by the arithmetic method Ra' (4) under each of new compositions is found in Mizumoto (1981a).

3.1 ~~The case of Rm' = (A x B) ∪ (7A x C) under max-min composition~~ "o"

Let A be a fuzzy set and R_1, R_2 be fuzzy relations, then, in general, the following identity holds for the max-min composition "o" (See (16)).

$$A \circ (R_1 \cup R_2) = (A \circ R_1) \cup (A \circ R_2)$$

Using this fact, (7) will be

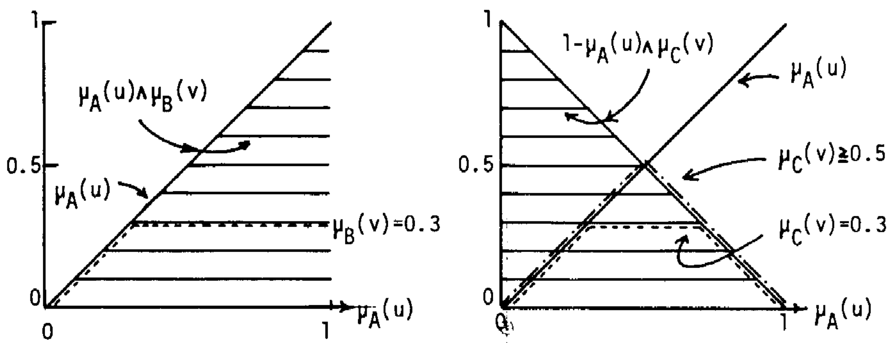
$$A' \circ [(A \times B) \cup (7A \times C)]$$
$$= [A' \circ (A \times B)] \cup [A' \circ (7A \times C)] \qquad (24)$$

Therefore, at A' = A, the membership function of A o (A x B) is given as

$$\mu_{Ao(AxB)}(v) = \bigvee_{x} [\mu_A(u) \wedge (\mu_A(u) \wedge \mu_B(v))]. \qquad (25)$$

If $\mu_B(v)$ is, say, 0.3, then the expression in [...] of (25) will be shown by the dotted line in Fig.1(a). The value $\mu_{Ao(AxB)}(v)$ at $\mu_B(v) = 0.3$ becomes 0.3 by taking maximum of this line by virtue of (25). In general, we can have for any $\mu_B(v)$

$$\mu_{Ao(AxB)}(v) = \mu_B(v). \qquad (26)$$



(a) $\mu_{Ao(AxB)}(v)$ (b) $\mu_{Ao(7AxC)}(v)$

Fig. 1 A o [(A x B) U (7A x C)]

On the other hands, the membership function of A o (7A x C) of (24) is as follows.

$$\mu_{Ao(7AxC)}(v) = \bigvee_{u} [\mu_A(u) \wedge (1-\mu_A(u) \wedge \mu_C(v))]. \qquad (27)$$

For example, at $\mu_C(v) = 0.3$ (≤ 0.5), the expression in [...] of (27) is shown by the dotted line "---" in Fig.1(b), and at $\mu_C(v) \geq 0.5$ it is shown by the line "_._.". Thus, from the figure

$$\mu_{Ao(7AxC)}(v) = \begin{cases} \mu_C(v) & \dots \ \mu_C(v) \leq 0.5 \\ 0.5 & \dots \ \mu_C(v) \geq 0.5 \end{cases}$$

~~Stated alternatively~~,

$$\mu_{Ao(7A\times C)}(v) = 0.5 \wedge \mu_C(v). \quad (28)$$

In the sequal, we can obtain the membership function of Dm = A o [(A x B) ∪ (7A x C)] as follows by taking max (v) of (26) and (28) by virtue of (24).

$$\mu_{Dm}(v) = \mu_B(v) \vee (0.5 \wedge \mu_C(v)) \quad \ldots \quad \text{at } A' = A \quad (29)$$

In the same way, we can have the consequence Dm at A' = <u>not</u> A (= 7A) under "o" as follows.

$$\mu_{Dm}(v) = \mu_C(v) \vee (0.5 \wedge \mu_B(v)) \quad \ldots \quad \text{at } A' = \underline{\text{not}}\ A \quad (30)$$

According to our intuition, it seems that the consequence D in (1) should be B and C, respectively, when A' is A and <u>not</u> A under the conditional proposition "If x is A then y is B else y is C." Namely, the following inference may be a quite natural demand.

Prem 1:	If x is A then y is B else y is C	
Prem 2:	x is A	(31)
Cons:	y is B	

Prem 1:	If x is A then y is B else y is C	
Prem 2:	x is not A	(32)
Cons:	y is C	

However, from the consequence Dm of (29) and (30) by the Maximin Rule Rm' at A' = A, <u>not</u> A, it is found that Dm is not equal to B at A' = A, and to C at A' = <u>not</u> A. <u>Namely</u>, the Maximin Rule Rm' under the max-min composition "o" does not <u>satisfy</u> the natural criteria of (31) and (32). Therefore, it seems that the Maximin Rule is not a suitable method for the fuzzy conditional inference with "If ... then ... else ...".

On the contrary, we shall next show that the Maximin Rule Rm' can satisfy the natural criteria when using the new compositions of "□" and "▲".

3.2 <u>The case of Rm' = (A x B) ∪ (7A x C) under max-Θ composition "□"</u>

The consequence Dm under max-Θ composition "□" is given as

$$\begin{aligned} Dm &= A \,\Box\, [(A \times B) \cup (7A \times C)] \\ &= [A \,\Box\, (A \times B)] \cup [A \,\Box\, (7A \times C)] \end{aligned} \quad (33)$$

The membership function of A □ (A x B) is

$$\mu_{A\Box(A\times B)}(v) = \bigvee_u \{\mu_A(u) \ominus (\mu_A(u) \wedge \mu_B(v))\}$$

$$= \bigvee_u \{0 \vee [\mu_A(u) + (\mu_A(u) \wedge \mu_B(v)) - 1]\}$$

$$= \bigvee_u \{0 \vee [(\mu_A(u) + \mu_A(u) - 1) \wedge (\mu_A(u) + \mu_B(v) - 1)]\}$$

$$= \bigvee_u \{0 \vee [(2\mu_A(u) - 1) \wedge (\mu_A(u) + \mu_B(v) - 1)]\} \quad (34)$$

Figure 2 show[illegible]rtial plots of the expression in {....} of (34) with $\mu_B(v)$ as parameter. When $\mu_B(v)$ is equal to, say, 0.7, the expression is indicated by the broken line, and thus (34) at $\mu_B(v) = 0.7$ is seen to be 0.7 by observing the maximum of this line. Therefore, in general, we can have $\mu_{A\Box(A\times B)}(v) = \mu_B(v)$ for any $\mu_B(v)$. Namely, we have

$$\mu_{A\Box(A\times B)}(v) = \mu_B(v). \qquad (35)$$

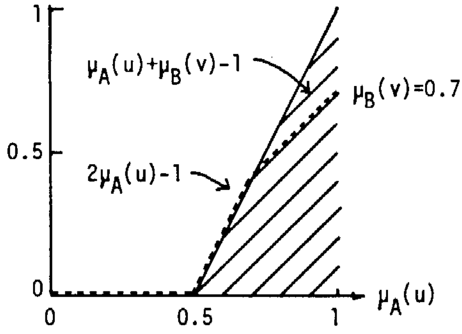


Fig. 2 $\mu_{A\Box(A\times B)}(v)$

On the other hand, the membership function of A □ (7A x C) in (33) is given by

$$\mu_{A\Box(7A\times C)}(v)$$

$$= \bigvee_u \{\mu_A(u) \odot (1 - \mu_A(u) \wedge \mu_C(v))\}$$

$$= \bigvee_u \{0 \vee [\mu_A(u) + (1 - \mu_A(u) \wedge \mu_C(v)) - 1]\}$$

$$= \bigvee_u \{0 \vee [0 \wedge (\mu_A(u) + \mu_C(v) - 1)]\}$$

$$= \bigvee_u 0 = 0$$

Thus, we have

$$\mu_{A\Box(7A\times C)}(v) = 0 \qquad (36)$$

Therefore, from (35) and (36) the consequence Dm is obtained as

$$\mu_{Dm}(v) = \mu_B(v) \vee 0 = \mu_B(v) \quad \ldots \quad \text{at } A' = A \qquad (37)$$

In the same way, the consequence Dm at A' = <u>not</u> A is given as

$$\mu_{Dm}(v) = \mu_C(v) \quad \ldots \quad \text{at } A' = \underline{\text{not}}\ A \qquad (38)$$

It is found from the results of (37) and (38) that the Maximin Rule Rm' can satisfy the reasonable criteria of (31) and (32) under the max-⊙ composition "□". Similarly, we can get the consequences Dm = B at A' = A, and Dm = C at A' = <u>not</u> A under max-▵ composition "▲".

5.3 <u>The case of Rb' = (7A x V ∪ U x B) ∩ (A x V ∪ U x C) under max-▵ composition</u>

Finally, we shall show the consequences Db by the Fuzzified Binary Rule Rb' of (5) under the max-▵ composition "▲". Note that the consequences Db under the max-min composition "o" is as follows (Mizumoto, 1982b).

$$Db = A \circ Rb'$$
$$= A \circ [(\neg A \times V \cup U \times B) \cap (A \times V \cup U \times C)]$$
$$= \int_V \mu_B(v) \vee 0.5 \,/v$$
$$\neq B$$

$$Db = \underline{\text{not}}\ A \circ Rb'$$
$$= \int_V \mu_C(v) \vee 0.5 \,/v$$
$$\neq C$$

which indicates the failure of the criteria of (31) and (32).

On the other hands, the consequence Db by Rb' under the max-$\dot{\wedge}$ composition "▲" is obtained as follows. The membership function of Db is given as

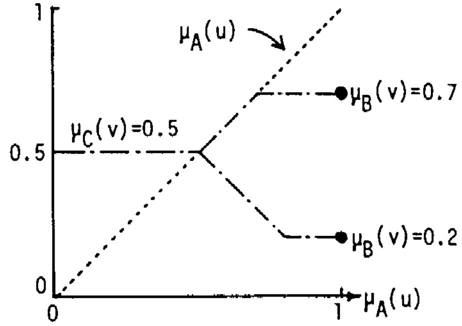


Fig. 3 $\mu_{Db}(v)$

$$\mu_{Db}(v) = \bigvee_u \{\mu_A(u) \mathbin{\dot{\wedge}} [(1 - \mu_A(u) \vee \mu_B(v)) \wedge (\mu_A(u) \vee \mu_C(v))]\}$$

where

$$x \mathbin{\dot{\wedge}} y = \begin{cases} x & \dots\ y = 1 \\ y & \dots\ x = 1 \\ 0 & \dots\ x,y < 1 \end{cases}$$

The expression $f(\mu_A(u)) = (1-\mu_A(u) \vee \mu_B(v)) \wedge (\mu_A(u) \vee \mu_C(v))$ with $\mu_B(v)$ and $\mu_C(v)$ as parameters is depicted partly in Fig.3 at $\mu_C(v) = 0.5$ and $\mu_B(v) = 0.7$ and 0.2. From this figure the expression $\mu_A(u) \mathbin{\dot{\wedge}} f(\mu_A(u))$ is shown by the black circle, that is,

$$\mu_A(u) \mathbin{\dot{\wedge}} f(\mu_A(u)) = \begin{cases} \mu_B(v) & \dots\ \mu_A(u) = 1 \\ 0 & \dots\ \text{otherwise} \end{cases}$$

Therefore, $\mu_{Db}(v)$ is

$$\mu_{Db}(v) = \bigvee_u \{\mu_A(u) \mathbin{\dot{\wedge}} f(\mu_A(u))\}$$
$$= \mu_B(v).$$

Thus, we have Db as

$$Db = B \quad \dots \quad \text{at } A' = A \tag{39}$$

which leads to the satisfaction of the criterion (31). In the same way we can get the consequence Db = C at A' = not A under the max-$\dot{\wedge}$ composition "▲".

We can obtain the consequences Dm, Da, Db and Dgg in the same way as the ways discussed above. Table 1 summarizes the consequences inferred by all the inference methods under max-min composition, max-⊙ composition and max-$\dot{\wedge}$ composition,

Table 1 Inference Results

Composition / Rule \ A'	Max-Min Composition "o"		Max-θ Composition "□"		Max-⩓ Composition "▲"	
	A	not A	A	not A	A	not A
Rm'	$\mu_B(v) \vee (0.5 \wedge \mu_C(v))$	$\mu_C(v) \vee (0.5 \wedge \mu_B(v))$	$\mu_B(v)$	$\mu_C(v)$	$\mu_B(v)$	$\mu_C(v)$
Ra'	$\frac{1 + \mu_B(v)}{2}$	$\frac{1 + \mu_C(v)}{2}$	$\mu_B(v)$	$\mu_C(v)$	$\mu_B(v)$	$\mu_C(v)$
Rb'	$\mu_B(v) \vee 0.5$	$\mu_C(v) \vee 0.5$	$\mu_B(v)$	$\mu_C(v)$	$\mu_B(v)$	$\mu_C(v)$
Rgg'	$\mu_B(v)$	$\mu_C(v)$	$\mu_B(v)$	$\mu_C(v)$	$\mu_B(v)$	$\mu_C(v)$

From this table it follows that all of the inference methods Rm', Ra', Rb' and Rgg' can satisfy the reasonable criteria (31) and (32) under the new compositions of max-Θ composition and max-Δ composition, though only the method Rgg' satisfies the criteria under the max-min composition. Therefore, all the methods except Rgg' are not suitable inference methods under the max-min composition which is usually used as the compositional rule of inference, but these methods are found to be suitable methods if max-Θ composition and max-Δ composition are used in the compositional rule of inference.

4. CONCLUSION

In this paper we have discussed only the case of A' = A and not A. It will be of interest to discuss the cases of A' = very A, more or less A, slightly A and so on, where A' is obtained by attaching to A various linguistic hedges such as very, more or less, slightly and so on (Mizumoto, 1983b; Ezawa, 1983).

REFERENCES

Ezawa, Y., and M. Mizumoto (1983). Linguistic hedges and reasonable fuzzy inference. Proc. of Symp. on Fuzzy Information, Knowledge Representation and Decision Analysis (Marseille, July 19-21, 1983), 235-240.

Fukami, S., M. Mizumoto, and K. Tanaka (1980). Some considerations on fuzzy conditional inferences. Fuzzy Sets and Systems, 4, 3, 243-273.

Mamdani, E.H. (1977). Application of fuzzy logic to approximate reasoning using linguistic systems. IEEE Trans. on Computer, c-26, 1182-1191.

Mizumoto, M., S. Fukami, and K. Tanaka (1979). Several methods for fuzzy conditional inferences. Proc. of IEEE Conf. on Decision and Control (Florida, Dec. 12-14, 1979), 777-782.

Mizumoto, M. (1981a). Note on the arithmetic rule by Zadeh for fuzzy conditional inference. Cybernetics and Systems, 12, 3, 247-306.

Mizumoto, M. (1981b). Fuzzy sets and their operations, I. Information and Control, 48, 1, 30-48.

Mizumoto, M. (1981c). Fuzzy sets and their operations,II. Information and Control, 50, 2, 160-174.

Mizumoto, M., and H.J. Zimmermann (1982a). Comparison of fuzzy reasoning methods. Fuzzy Sets and Systems, 8, 3, 253-283.

Mizumoto, M. (1982b). Fuzzy reasoning with a fuzzy conditional proposition "If ... then ... else ...". In R.R. Yager (ed.), Fuzzy Sets and Possibility Theory: Recent Development. Pergamon Press, 211-223.

Mizumoto, M. (1982c). Fuzzy conditional inference under max-Θ composition. Information Sciences, 27, 2, 183-209.

Mizumoto, M. (1982d). Fuzzy inference using max-Δ composition in the compositional rule of inference. In M.M. Gupta and E. Sanchez (eds.), Approximate Reasoning in Decision Analysis. North-Holland, Amsterdam, 67-76.

Mizumoto, M. (1983a). Fuzzy reasoning under new compositional rules of inference. Proc. of 13th Int. Symp. on Multiple-Valued Logic (Kyoto, May 23-25, 1983), 273-278.

Mizumoto, M. (1983b). Fuzzy reasoning with various fuzzy inputs. Proc. of Symp. on Fuzzy Information, Knowledge Representation and Decision Analysis (Marseille, July 19-21, 1983), 153-158.

Rescher, N. (1969). Many Valued Logic. MacGraw-Hill, New York.

Zadeh, L.A. (1975). Calculus of fuzzy restriction. In L.A. Zadeh et al. (eds.), Fuzzy Sets and Their Applications to Cognitive and Decision Precesses, Academic Press, New York, 1-39.