FUZZY REASONING WITH VARIOUS FUZZY INPUTS

M. Mizumoto

Department of Management Engineering, Osaka Electro-Communication University, Neyagawa, Osaka 572, Japan

Abstract. This paper compares inference results of a fuzzy conditional inference with a fuzzy input A' and a fuzzy conditional A ⟹ B under several translating rules for the conditional A ⟹ B by Zadeh, Mamdani and Mizumoto when a fuzzy input A' is a fuzzy set obtained by attaching to the fuzzy set A a linguistic hedge such as *slightly, sort of, highly* and so on. It is shown that the translating rule Rs proposed before by the author can get reasonable inference results which fit our intuition. Moreover, a new composition called "max-$\triangle$ composition" is introduced and it is shown that the inference results for various fuzzy inputs A' are better than those under the ordinal compositional rule of inference which uses "max-min composition."

Keywords. Fuzzy reasoning; fuzzy conditional inference; linguistic hedge

INTRODUCTION

In our daily life we often make such an inference of the form:

Prem 1:	If x is A then y is B	
Prem 2:	x is A'	(1)
Cons:	y is B'	

where A, A', B, B' are fuzzy concepts. In order to make such an inference, Zadeh (1975) suggested an inference rule called "compositional rule of inference" which infers B' of Cons from Prem 1 and 2 by taking the max-min composition of fuzzy set A' and the fuzzy relation which is translated from the fuzzy conditional proposition "If x is A then y is B." In this connection, he (1975), Mamdani (1977) and Mizumoto et al. (1979, 1982) suggested several translating rules for translating the fuzzy proposition "If x is A then y is B" into a fuzzy relation.

In Mizumoto (1979, 1982) we compared inference results by their translating rules only when A' of Prem 2 is A, *very* A ($= A^2$), *more or less* A ($= A^{0.5}$) and *not* A ($= \neg A$), most of which are special case of A^{α}.

It will be of interest to obtain and discuss inference results under other kinds of A'. In this paper we obtain inference results when A' is a fuzzy set obtained by attaching to the fuzzy set A a linguistic hedge such as *slightly, sort of, highly*, INT, WEAK, MIDI and so on, and discuss which translating rule can get reasonable inference results.

FUZZY CONDITIONAL INFERENCE

We shall consider the form of inference of (1) in which a fuzzy conditional proposition "If x is A then y is B" is contained. The inference may be viewed as fuzzy modus ponens which reduces to the classical modus ponens when A' = A and B' = B.

For simplicity, we shall rewrite (1) as

$$\begin{array}{l} A \Rightarrow B \\ A' \\ \hline B' \end{array} \qquad (2)$$

where A, A', B, B' are fuzzy concepts which are represented by fuzzy sets in universes of discourse U, U, V and V, respectively.

The fuzzy conditional A ⟹ B may represent a certain relationship between A and B. From this point of view, Zadeh (1975), Mamdani (1977) and Mizumoto et al. (1979, 1982) proposed several translating rules for translating A ⟹ B into a fuzzy relation in U x V.

Let A and B be fuzzy sets in U and V, respectively, and let x and ⊕ be cartesian product and bounded-sum for fuzzy sets, respectively. Then the following fuzzy relations in U x V can be translated from A ⟹ B. The fuzzy relations Rm and Ra were proposed by Zadeh, Rc by Mamdani, and the others by Mizumoto by introducing the implications of many-valued logic systems. For example, Ra (arithmetic rule) is given as

$$Ra = (\neg A \times V) \oplus (U \times B) \qquad (3)$$
$$= \int_{U \times V} 1 \wedge (1 - \mu_A(u) + \mu_B(v)) \; / \; (u,v).$$

It is noted that this rule is based on the implication rule of Lukasiewicz's logic, i.e.,

$$a \to b = 1 \wedge (1 - a + b), \quad a,b \in [0,1] \qquad (4)$$

Therefore, as other translating rules, it is possible to introduce other implication rules of many-valued logic systems to a translating rule for A ⟹ B (cf. Mizumoto (1982)).

Now, let $\mu_A(u) = a$ and $\mu_B(v) = b$, then we have such translating rules as

Rm: $(a \wedge b) \vee (1-a)$. (5)

Ra: $1 \wedge (1-a+b)$. (6)

Rc: $a \wedge b$. (7)

Rs: $\begin{cases} 1 & \dots\ a \leq b, \\ 0 & \dots\ a > b. \end{cases}$ (8)

Rg: $\begin{cases} 1 & \dots\ a \leq b, \\ b & \dots\ a > b. \end{cases}$ (9)

Rb: $(1-a) \vee b$. (10)

R_Δ: $\begin{cases} 1 & \dots\ a \leq b, \\ \frac{b}{a} & \dots\ a > b. \end{cases}$ (11)

In the fuzzy modus ponens of (2), the consequence B' can be deduced from Prem 1 and 2 by taking the max-min composition "o" of the fuzzy set A' and the fuzzy relation obtained in (5)-(11) (the compositional rule of inference). For example, the consequence Ba' by the rule Ra is given as

$$Ba' = A' \circ Ra. \quad (12)$$

$$\mu_{Ba'}(v) = \bigvee_u \{\mu_{A'}(u) \wedge \mu_{Ra}(u,v)\} \quad (13)$$
$$= \bigvee_u \{\mu_{A'}(u) \wedge [1 \wedge (1-\mu_A(u)+\mu_B(v))]\}.$$

In the same way, we have

$$Bm' = A' \circ Rm. \quad (14)$$
$$Bc' = A' \circ Rc.$$
$$\vdots$$

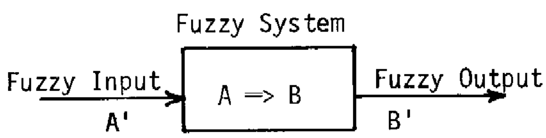

Fig. 1. Fuzzy system (A => B) with fuzzy input A' and fuzzy output B'.

The fuzzy modus ponens of (2) represents that the consequence B' is deduced when the premise A' is given under the condition A => B. If we regard the fuzzy conditional A => B (that is, fuzzy relation) as a fuzzy system, then A' and B' correspond to "fuzzy input" and "fuzzy output," respectively (See Fig.1). It will be interesting to discuss what kinds of fuzzy outputs B' are obtained when various kinds of fuzzy inputs A' are input to the fuzzy system.

LINGUISTIC HEDGES

In order to obtain various fuzzy inputs A', we shall briefly review some linguistic hedges proposed by Zadeh (1975) and introduce new artificial linguistic hedges.

Let A be a fuzzy set in U. Linguistic hedges which act on the fuzzy set A are listed as follows (See Fig.2).

As a special case of A^α $(=\int_U \mu_A(u)^\alpha/u)$, we can have such linguistic hedges as

$$\text{CON}(A) = \textit{very } A = A^2 \quad (15)$$
$$\text{DIL}(A) = \textit{more or less } A = A^{0.5} \quad (16)$$
$$\textit{minus } A = A^{0.75} \quad (17)$$
$$\textit{plus } A = A^{1.25} \quad (18)$$
$$\textit{highly } A = \textit{plus very } A = A^{2.5} \quad (19)$$

where CON, DIL and the following INT stand for "concentration", "dilation" and "contrast intensification", respectively.

$$\text{INT}(A) = \int_{\mu_A(u)\leq 0.5} 2\mu_A(u)^2/u + \int_{\mu_A(u)\geq 0.5} 1-2(1-\mu_A(u))^2/u \quad (20)$$

Using the above linguistic hedges, we can obtain the following linguistic hedges.

$$\textit{slightly } A = \text{NORM}(A \textit{ and not very } A)^{¶}$$
$$= \text{NORM}(A \cap \neg\text{CON}(A)) \quad (21)$$
$$= \frac{\sqrt{5}-1}{2}\left(\int_U \mu_A(u) \wedge (1-\mu_A(u)^2)/u\right).$$

$$\textit{sort of } A = \text{NORM}(\text{DIL}(A) \cap \neg\text{CON}(A)^2)$$
$$= \text{NORM}(\textit{more or less but not very very } A) \quad (22)$$
$$\fallingdotseq 1.232\left(\int_U \mu_A(u)^{0.5} \wedge (1-\mu_A(u)^4)/u\right).$$

The above are main linguistic hedges proposed by Zadeh. It is found that linguistic hedges can be viewed as operators which act on a fuzzy set. From this point of view, we can introduce new operators on a fuzzy set. Some of these are introduced as follows.

The effect of "contrast weakening" (WEAK, for short) is the opposite of that of INT.

$$\text{WEAK}(A) \quad (23)$$
$$= \int_{\mu_A(u)\leq 0.5} -2(\mu_A(u)^2-\mu_A(u))/u + \int_{\mu_A(u)\geq 0.5} 2(\mu_A(u)-\tfrac{1}{2})^2+\tfrac{1}{2}/u$$

The operator of "middle intensification" (MIDI for short) has the effect of intensifing middle grades and is defined as

$$\text{MIDI}(A) = \text{NORM}(A \cap \neg A) = 2(A \cap \neg A)$$
$$= \int_U 2\mu_A(u) \wedge 2(1-\mu_A(u))/u. \quad (24)$$

As the opposite operator to MIDI, we can give MIDW ("middle weakening") as follows.

$$\text{MIDW}(A) = \neg\text{MIDI}(A) = \neg 2(A \cap \neg A)$$
$$= \int_U (1-2\mu_A(u)) \wedge (2\mu_A(u)-1)/u. \quad (25)$$

The operator αCUT obtains the α-level set of a fuzzy set A. The operator αCUT* is the opposite operator to αCUT, that is,

$$\alpha\text{CUT}(A) = \int_{\mu_A(u)\geq\alpha} 1/u + \int_{\mu_A(u)<\alpha} 0/u \quad (26)$$

¶ NORM (= normalization) is defined as

$$\text{NORM}(A) = \frac{1}{\mu_{A*}} A \quad \text{with } \mu_{A*} = \bigvee_u \mu_A(u)$$

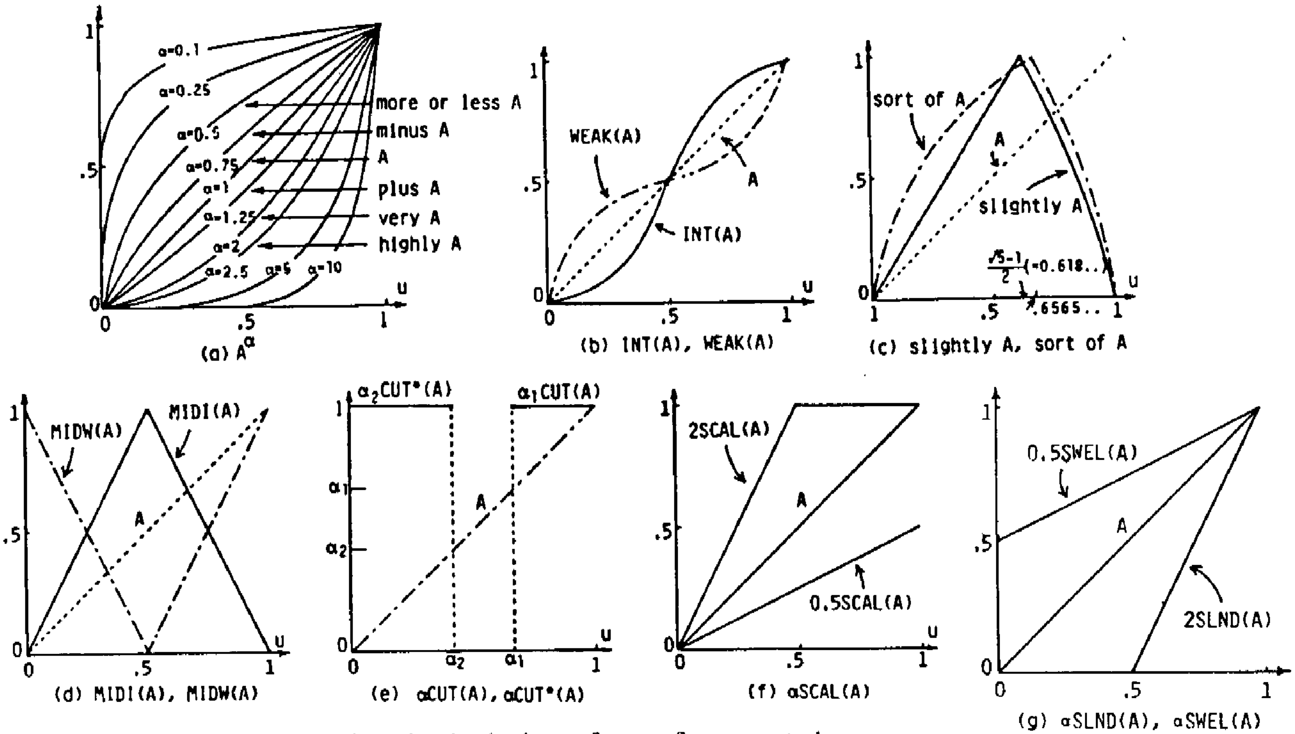

Fig.2. Various linguistic hedges for a fuzzy set A

$$\alpha CUT^*(A) = \int_{\mu_A(u)\leq\alpha} 1/u + \int_{\mu_A(u)>\alpha} 0/u \tag{27}$$

The operator αSCAL which gives the scalor product αA of α and A is defined as

$$\alpha SCAL(A) = \int_U \alpha\mu_A(u) \wedge 1 \ /u \tag{28}$$

Finally we shall give two operators which have the effects of "slenderizing" and "swelling" a fuzzy set A. The first is named as αSLND and the latter as αSWEL. They have the same expression but they are distinguished from the values of their parameter α. That is to say,

$$\alpha SLND(A) = \int_U 0 \vee (\alpha\mu_A(u) + 1 - \alpha)/u \ \ldots \ \alpha\geq 1 \tag{29}$$

$$\alpha SWEL(A) = \int_U \alpha\mu_A(u) + 1 - \alpha \ /u \ \ldots \ \alpha\leq 1 \tag{30}$$

Fig. 2 shows the effects of the linguistic hedges of (15)-(30) on a fuzzy set A, where A is a fuzzy set $\int_U u/u$ in U = [0,1].

INFERENCE RESULTS FOR VARIOUS FUZZY INPUTS

We shall obtain the consequence B' under each translating rule of (5)-(11) when A' is a fuzzy set given by applying linguistic hedges to A, and discuss which method can get reasonable consequences.

We shall discuss only the case of Rm (5) at $A' = A^\alpha$ (as a general case of (15)-(19)).

When $A' = A^\alpha$, the consequence Bm' is obtained as

$$\mu_{Bm'}(v) = \bigvee_u \{\mu_A(u)^\alpha \wedge [(\mu_A(u) \wedge \mu_B(v)) \vee (1-\mu_A(u))]\}$$

This expression can be rewritten as (32) by letting

$$x = \mu_A(u), \quad b = \mu_B(v), \quad bm' = \mu_{Bm'}(v) \tag{31}$$

under the assumption that $\mu_A(u)$ takes all values in [0,1] according to u varying all over U, that is, μ_A is a function onto [0,1], i.e., x is on [0,1].

$$bm' = \bigvee_x \{x^\alpha \wedge [(x \wedge b) \vee (1-x)]\} \ ; \tag{32}$$

$$f(x) = x^\alpha \wedge [(x \wedge b) \vee (1-x)] \tag{33}$$

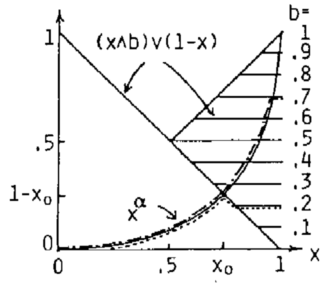

Fig.3 The way of obtaining (32)

Fig. 3 shows the expressions x^α and $(x \wedge b) \vee (1-x)$ using a parameter b. When x^α is as in this figure and b is equal to, say, 0.2, the expression f(x) is indicated by the broken line and hence bm' at b = 0.2 is the maximum value of this broken line. The value is equal to the height of the cross point of x^α and 1-x. Thus, let x_o ($\in$ [0,1]) be the solution of $x^\alpha = 1-x$, then the height (i.e., maximum value) is given by $1 - x_o$. Therefore, we have bm' at $b \leq 1-x_o$ as

$$bm' = \bigvee_x f(x) = 1 - x_o \ \ldots \ b \leq 1 - x_o$$

On the other hand, when b = 0.7 ($\geq 1-x_o$), f(x) is given by the dot-dash line and its maximum value is b (=0.7). Thus, bm' = b for $b \geq 1-x_o$. Therefore, we have

$$bm' = \begin{cases} 1 - x_o & \ldots \ b \leq 1 - x_o \\ b & \ldots \ b \geq 1 - x_o \end{cases} = (1 - x_o) \vee b.$$

TABLE 1 Inference Results $B' = A' \circ R$ under Max-Min Composition "o"

A' \ A ⇒ B	Rm	Ra	Rc	Rs	Rg	Rb	R_Δ
A^α	$(1-x_0) \vee \mu_B$ (*)	$1 - x' + \mu_B$ (**)	μ_B	$\mu_B{}^\alpha$	$\begin{cases}\mu_B{}^\alpha & \ldots \alpha \leq 1 \\ \mu_B & \ldots \alpha \geq 1\end{cases}$	$(1-x_0) \vee \mu_B$	$\mu_B{}^{\frac{\alpha}{\alpha+1}}$
INT(A)	$0.5 \vee \mu_B$	$\frac{4\mu_B - 1 + \sqrt{9 - 8\mu_B}}{4}$	μ_B	INT(B)	$\begin{cases}\mu_B & \ldots 0 \leq \mu_B \leq 0.5 \\ 1-2(1-\mu_B)^2 & \ldots 0.5 \leq \mu_B \leq 1\end{cases}$	$0.5 \vee \mu_B$	See (35)
slightly A	$(\frac{\sqrt{5}-1}{2} \vee \mu_B) \wedge m_0$ (***)	$\frac{\sqrt{5}-1}{2}(1+\mu_B) \wedge 1$	$\mu_B \wedge m_0$	$\frac{\sqrt{5}+1}{2}\mu_B \wedge 1$	$\frac{\sqrt{5}+1}{2}\mu_B \wedge 1$	$\frac{\sqrt{5}-1}{2} \vee \mu_B$	$\sqrt{\frac{\sqrt{5}+1}{2}\mu_B} \wedge 1$
sort of A	$(0.689 \vee \mu_B) \wedge 0.779$	$\frac{c(\sqrt{4(1+\mu_B)+c^2}-c)}{2} \wedge 1$ (****)	$\mu_B \wedge 0.779$	$c\sqrt{\mu_B} \wedge 1$	$c\sqrt{\mu_B} \wedge 1$	$0.689 \vee \mu_B$	$\sqrt[3]{c^2\mu_B} \wedge 1$
WEAK(A)	$0.5 \vee \mu_B$	$\frac{3 + 4\mu_B - \sqrt{1 + 8\mu_B}}{4}$	μ_B	WEAK(B)	$\begin{cases}-2(\mu_B{}^2 - \mu_B) & \ldots 0 \leq \mu_B \leq 0.5 \\ \mu_B & \ldots 0.5 \leq \mu_B \leq 1\end{cases}$	$0.5 \vee \mu_B$	See (36)
MIDI(A)	$\frac{2}{3}$	$\frac{2}{3}(1+\mu_B) \wedge 1$	$\mu_B \wedge \frac{2}{3}$	$2\mu_B \wedge 1$	$2\mu_B \wedge 1$	$\frac{2}{3} \vee \mu_B$	$\sqrt{2\mu_B} \wedge 1$
MIDW(A)	1	1	$\frac{1}{3} \vee \mu_B$	1	1	1	1
αCUT(A)	$(1-\alpha) \vee \mu_B$	$(1 - \alpha + \mu_B) \wedge 1$	μ_B	αCUT(B)	$\begin{cases}1 & \ldots \mu_B \geq \alpha \\ \mu_B & \ldots \mu_B < \alpha\end{cases}$	$(1-\alpha) \vee \mu_B$	$\frac{\mu_B}{\alpha} \wedge 1$
αCUT*(A)	1	1	$\mu_B \wedge \alpha$	1	1	1	1
αSCAL(A)	$\begin{cases}(\frac{\alpha}{\alpha+1} \vee \mu_B) \wedge \alpha & \ldots \alpha \leq 1 \\ \frac{\alpha}{\alpha+1} \vee \mu_B & \ldots \alpha \geq 1\end{cases}$	$\begin{cases}\frac{\alpha}{\alpha+1}(1+\mu_B) \wedge \alpha & \ldots \alpha \leq 1 \\ \frac{\alpha}{\alpha+1}(1+\mu_B) \wedge 1 & \ldots \alpha \geq 1\end{cases}$	$\begin{cases}\mu_B \wedge \alpha & \ldots \alpha \leq 1 \\ \mu_B & \ldots \alpha \geq 1\end{cases}$	αSCAL(B)	$\begin{cases}\mu_B \wedge \alpha & \ldots \alpha \leq 1 \\ \alpha\mu_B \wedge 1 & \ldots \alpha \geq 1\end{cases}$	$\begin{cases}(\frac{\alpha}{\alpha+1} \vee \mu_B) \wedge \alpha & \ldots \alpha \leq 1 \\ \frac{\alpha}{\alpha+1} \vee \mu_B & \ldots \alpha \geq 1\end{cases}$	$\begin{cases}\sqrt{\alpha\mu_B} \wedge \alpha & \ldots \alpha \leq 1 \\ \sqrt{\alpha\mu_B} \wedge 1 & \ldots \alpha \geq 1\end{cases}$
αSLND(A) ($\alpha \geq 1$) αSWEL(A) ($\alpha \leq 1$)	$\frac{1}{1+\alpha} \vee \mu_B$	$\frac{1+\alpha\mu_B}{1+\alpha}$	μ_B	αSLND(B) αSWEL(B)	μ_B αSWEL(B)	$\frac{1}{1+\alpha} \vee \mu_B$	$\frac{1 - \alpha + \sqrt{(1-\alpha)^2 + 4\alpha\mu_B}}{2}$

(*) x_0 ($\in [0,1]$) is the solution of $x^\alpha = 1 - x$

(**) x' ($\in [0,1]$) is the solution of $x^\alpha = 1 - x + \mu_B$

(***) $m_0 = \frac{1 - \sqrt{5} + \sqrt{22 - 2\sqrt{5}}}{4} = 0.7376\ldots$

(****) $c = 1.232\ldots$

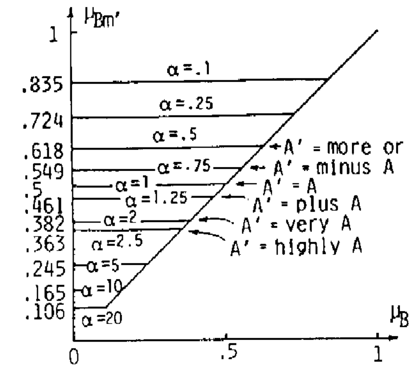

Fig.4. Bm' = A^{α} o Rm

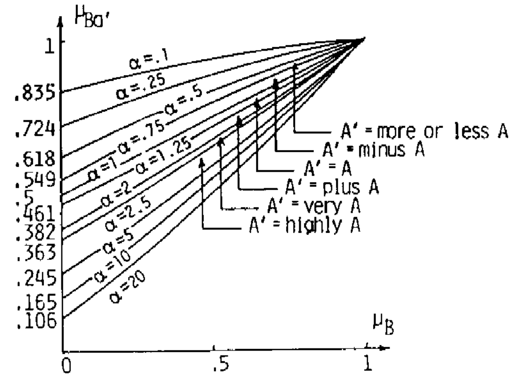

Fig.5 Ba' = A^{α} o Ra

This result is shown to hold for any α. Therefore, using the notation of (31), the consequence Bm' = A^{α} o Rm is as follows.

$$\mu_{Bm'}(u) = (1 - x_o) \vee \mu_B(u) \qquad (34)$$

where x_0 ($\epsilon[0,1]$) is the solution of $x^{\alpha} = 1 - x$.

Fig.4 shows the consequence Bm' (= A^{α} o Rm) using a parameter α. Fig.5 also indicated Ba' (= A^{α} o Ra) which can be obtained in the same way as Bm'. Table 1 shows the inference results for other rules and A', where the notation μ_B stands for $\mu_B(v)$.

The consequences BΔ' for the rule R_Δ (11) at A' = INT(A) and WEAK(A) are shown in (35) and (36).

Case of BΔ' = INT(A) o RΔ:

$$\mu_{B_\Delta'} = \begin{cases} \sqrt[3]{2\mu_B^{\,2}} & \dots \quad 0 \leq \mu_B \leq \frac{1}{4} \\ \frac{\mu_B}{x} & \dots \quad \frac{1}{4} \leq \mu_B \leq 1 \end{cases} \qquad (35)$$

where

$$x = \frac{1}{3}(2 + \sqrt{10}\cos\frac{\theta+\pi}{3}), \quad \theta = \cos^{-1}[\frac{\sqrt{10}}{50}(27\mu_B - 14)].$$

Case of BΔ' = WEAK(A) o RΔ:

$$\mu_{B_\Delta'} = \begin{cases} \frac{\mu_B}{x} & \dots \quad 0 \leq \mu_B \leq \frac{1}{4} \\ \frac{\mu_B}{x'} & \dots \quad \frac{1}{4} \leq \mu_B \leq 1 \end{cases} \qquad (36)$$

$$x = \frac{1}{3}(1 - 2\cos\frac{\psi+\pi}{3}), \quad \psi = \cos^{-1}(1 - \frac{27}{4}\mu_B),$$

$$x' = \frac{1}{3}\{1 + \sqrt[3]{\frac{1}{4}(27\mu_B - 5 + 9\sqrt{\frac{27\mu_B^{\,2} - 10\mu_B + 1}{3}})}$$

$$+ \sqrt[3]{\frac{1}{4}(27\mu_B - 5 - 9\sqrt{\frac{27\mu_B^{\,2} - 10\mu_B + 1}{3}})}\;\}$$

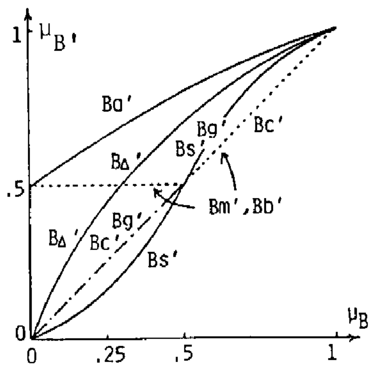

Fig.6. B' = INT(A) o R

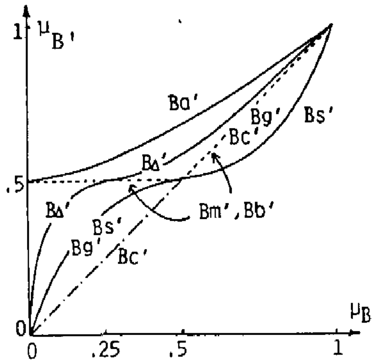

Fig.7. B' = WEAK(A) o R

The membership function $\mu_{B\Delta'}$ of (35) and (36) are very complicated and so we shall show in Fig. 6 and 7 the diagrams of $\mu_{B\Delta'}$, together with the inference results by other methods.

In the form of the fuzzy inference of (2), it is quite natural to expect that B' = B will be obtained when A' = A (satisfaction of "modus ponens"). This criterion is satisfied by the rules Rc, Rs and Rg. See the results for A^{α} at α = 1 in Table 1. Namely, these methods obtain A o R = B. Moreover, it is also natural to expect B' ≃ B when A' ≃ A. The method Rs satisfies this criterion and obtains consequences B' = B^{α}, INT(B), WEAK(B), αCUT(B), αSCAL(B), αSLND(B) and αSWEL(B) at A' = A^{α}, INT(A), WEAK (A), αCUT(A), αSCAL(A), αSLND(A) and αSWEL(A), respectively. The other methods do not obtain such results. Note that Rc gets always B except the case of αSCAL(A), α ≤ 1. In Mizumoto (1982) we showed that all the methods except Rc satisfy the natural criterion that B' = *unknown* (= $\int_V 1/v$) is obtained at A' = *not* A. In this connection, for the criterion that B' ≃ *unknown* is obtained at A' ≃ *not* A, we may say that all the methods except Rc satisfy this criterion. In fact, these methods obtain B' = *unknown* at A' = MIDW(A) and αCUT*(A) which are similar to *not* A. Finally, it is not yet known what kinds of consequences are good for A' = *sort of* A, *slightly* A and MIDI(A) which lie between A and *not* A.

From the above considerations, we can conclude that the method Rs (8) is most suitable for the fuzzy conditional inference, though the given criteria are intuitive and rough.

INFERENCE RESULTS UNDER NEW COMPOSITION

We shall introduce new composition called "max -▵ composition" for the compositional rule of inference, and show that the inference results under the new composition are better than those under the max-min composition "o" discussed above.

Introducing new operation ▵ (drastic product)

$$x \mathbin{\triangle} y = \begin{cases} x & \dots \quad y = 1 \\ y & \dots \quad x = 1 \\ 0 & \dots \quad \text{otherwise} \end{cases} \qquad (37)$$

new composition "▲" is obtained from (13) by replacing ∧ by ▵. For example, the consequence Ba' by Ra under ▲ is given by the following.

TABLE 2 Inference Results $B' = A' \blacktriangle R$ under Max-$\dot{\wedge}$ Composition "$\blacktriangle$"

A' \ A⇒B	Rm	Ra	Rc	Rs	Rg	Rb	R_Δ
A^α	μ_B	$\begin{cases}\mu_B^\alpha & \ldots\alpha\le 1\\ \mu_B & \ldots\alpha\ge 1\end{cases}$	μ_B	B^α	$\begin{cases}\mu_B^\alpha & \ldots\alpha\le 1\\ \mu_B & \ldots\alpha\ge 1\end{cases}$	μ_B	$\begin{cases}\mu_B^\alpha & \ldots\alpha\le 1\\ \mu_B & \ldots\alpha\ge 1\end{cases}$
INT(A)	μ_B	$\mu_B \vee [1-2(1-\mu_B)^2]$	μ_B	INT(B)	$\mu_B \vee [1-2(1-\mu_B)^2]$	μ_B	$\mu_B \vee [1-2(1-\mu_B)^2]$
slightly A	$(\frac{3-\sqrt{5}}{2} \vee \mu_B) \wedge \frac{\sqrt{5}-1}{2}$	$(\frac{3-\sqrt{5}}{2} + \mu_B) \wedge 1$	$\mu_B \wedge \frac{\sqrt{5}-1}{2}$	$\frac{\sqrt{5}+1}{2}\mu_B \wedge 1$	$\frac{\sqrt{5}+1}{2}\mu_B \wedge 1$	$\frac{3-\sqrt{5}}{2} \vee \mu_B$	$\frac{\sqrt{5}+1}{2}\mu_B \wedge 1$
sort of A	$(0.3413 \vee \mu_B) \wedge 0.6587$	$c\sqrt{\mu_B} \wedge 1$ (*)	$\mu_B \wedge 0.6587$	$c\sqrt{\mu_B} \wedge 1$	$c\sqrt{\mu_B} \wedge 1$	$0.3413 \vee \mu_B$	$c\sqrt{\mu_B} \wedge 1$
WEAK(A)	μ_B	$-2(\mu_B^2 - \mu_B) \vee \mu_B$	μ_B	WEAK(B)	$-2(\mu_B^2 - \mu_B) \vee \mu_B$	μ_B	$-2(\mu_B^2 - \mu_B) \vee \mu_B$
MIDI(A)	0.5	$1 \wedge (\mu_B + 0.5)$	$\mu_B \wedge 0.5$	$2\mu_B \wedge 1$	$2\mu_B \wedge 1$	$0.5 \vee \mu_B$	$2\mu_B \wedge 1$
MIDW(A)	1	1	μ_B	1	1	1	1
αCUT(A)	$(1-\alpha) \vee \mu_B$	$(1-\alpha+\mu_B) \wedge 1$	μ_B	αCUT(B)	$\begin{cases}1 & \ldots\mu_B \ge \alpha\\ \mu_B & \ldots\mu_B < \alpha\end{cases}$	$(1-\alpha) \vee \mu_B$	$\frac{\mu_B}{\alpha} \wedge 1$
αCUT*(A)	1	1	$\mu_B \wedge \alpha$	1	1	1	1
αSCAL(A)	$\begin{cases}0 & \ldots\alpha<1\\ (1-\frac{1}{\alpha}) \vee \mu_B & \ldots\alpha\ge 1\end{cases}$	$\begin{cases}\alpha\mu_B & \ldots\alpha\le 1\\ 1 \wedge (1-\frac{1}{\alpha}+\mu_B) & \ldots\alpha\ge 1\end{cases}$	$\begin{cases}0 & \ldots\alpha<1\\ \mu_B & \ldots\alpha\ge 1\end{cases}$	αSCAL(B)	αSCAL(B)	$\begin{cases}0 & \ldots\alpha<1\\ (1-\frac{1}{\alpha}) \vee \mu_B & \ldots\alpha\ge 1\end{cases}$	αSCAL(B)
αSLND(A) ($\alpha \ge 1$)	μ_B	μ_B	μ_B	αSLND(B)	μ_B	μ_B	μ_B
αSWEL(A) ($\alpha \le 1$)	$(1-\alpha) \vee \mu_B$	αSWEL(B)	μ_B	αSWEL(B)	αSWEL(B)	$(1-\alpha) \vee \mu_B$	αSWEL(B)

(*) c = 1.232...

$$Ba' = A' \blacktriangle Ra$$

$$\mu_{Ba'}(v) = \bigvee_u \{\mu_{A'}(u) \;\dot{\wedge}\; \mu_{Ra}(u,v)\}$$

The same way is applicable to other translating rule of (5)-(11).

Table 2 lists the inference results by all the translating rules for various fuzzy premises A' under the max-$\dot{\wedge}$ composition. It is found from the results at $A' = A^\alpha$ with $\alpha = 1$ that all the translating rules can satisfy so called modus ponens under the max-$\dot{\wedge}$ composition, though only the rules Rc, Rs and Rg satisfy the modus ponens under the max-min composition as shown in Table 1. As for other fuzzy premises A', we shall consider the case of A' = WEAK(A). The inference results for A' = WEAK(A) under the max-min composition "o" and the max-$\dot{\wedge}$ composition "$\blacktriangle$" are found in Figs 7 and 8, respectively. The rule Rs infers Bs' = WEAK(B) under each of these compositions. The other rules do not get such results. But these rules under the max-$\dot{\wedge}$ composition can infer the consequences which are very similar to WEAK(B) as in Fig.8, which leads to the satisfaction of the criterion that $B' \simeq B$ at $A' \simeq A$. Such tendency can be observed for other A'. Therefore, we may say that the max-$\dot{\wedge}$ composition is a better compositional rule of inference than the max-min composition.

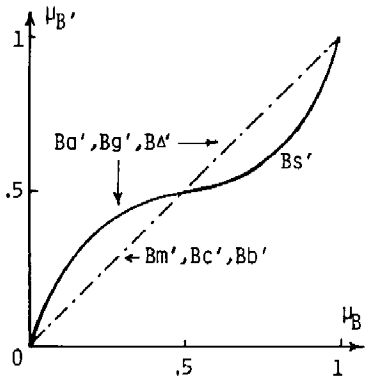

Fig.8. $B' = WEAK(A) \blacktriangle R$

CONCLUSION

Under the criterion that $B' \simeq B$ at $A' \simeq A$ and $B' \simeq$ *unknown* at $A' \simeq$ *not* A for the fuzzy modus ponens (1) and (2), it will be possible to make a quantitative analysis of the goodness of each translating rule by measuring a similarity of B' and B (or *unknown*) when A' is given which is similar to A (or *not* A).

The results of this paper will be useful to the problems such as fuzzy control, fuzzy diagnosis, fuzzy production system and so on which use fuzzy reasoning method with various fuzzy inputs.

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