Fuzzy Inferences with Various Fuzzy Inputs

M. Mizumoto

(Osaka Electro-Communication University, Neyagawa, Osaka 572, Japan)

1. Introduction

This paper compares inference results for a fuzzy conditional inference of the form:

$$\begin{array}{l} A \Rightarrow B \\ A' \\ \hline B' \end{array} \qquad A, A', B, B' \text{ : Fuzzy sets}$$

under several translating rules for a fuzzy conditional $A \Rightarrow B$ by Zadeh, Mamdani and Mizumoto when a fuzzy premise A' is a fuzzy set obtained by attaching to the fuzzy set A a linguistic hedge such as slightly, sort of, highly and so on. It is shown that the translating rule Rs proposed by the author can get quite reasonable inference results which fit our intuition.

2. Linguistic Hedges

We shall briefly review some linguistic hedges proposed by Zadeh[1] and introduce new artificial linguistic hedges.

Let A be a fuzzy set in a universe of discourse U. Linguistic hedges which act on the fuzzy set A are listed as follows (See Fig. 1).

As a special case of $A^{\alpha} (= \int_U \mu_A(u)^{\alpha}/u)$, we can have such linguistic hedges as

$$\text{CON}(A) = \text{very } A = A^2 \tag{1}$$

$$\text{DIL}(A) = \text{more or less } A = A^{0.5} \tag{2}$$

$$\text{minus } A = A^{0.75} \tag{3}$$

$$\text{plus } A = A^{1.25} \tag{4}$$

$$\text{highly } A = \text{plus very } A = A^{2.5} \tag{5}$$

本文于 1982 年 3 月 15 日收到.

where CON, DIL, and the following INT and NORM stand for "concentration", "dilation", "contrast intensification" and "normalization", respectively.

$$\mathrm{INT}(A) = \int_U \mu_{\mathrm{INT}(A)}(u)/u \tag{6}$$

where

$$\mu_{\mathrm{INT}(A)}(u) = \begin{cases} 2\mu_A(u)^2 & \cdots\ 0 \leqslant \mu_A(u) \leqslant 0.5 \\ 1 - 2(1-\mu_A(u))^2 & \cdots\ 0.5 \leqslant \mu_A(u) \leqslant 1 \end{cases}$$

$$\mathrm{NORM}(A) = \frac{1}{\mu_A^*} A \text{ with } \mu_A^* = \max_u \mu_A(u)$$

Using the above linguistic hedges, we can obtain the following linguistic hedges·

$$\begin{aligned} \text{slightly } A &= \mathrm{NORM}(A \cap \urcorner \mathrm{CON}(A)) \\ &= \mathrm{NORM}(A \text{ and not very } A) \\ &= \frac{\sqrt{5}-1}{2}\left(\int_U \mu_A(u) \wedge (1-\mu_A(u))^2/u\right). \end{aligned} \tag{7}$$

$$\begin{aligned} \text{sort of } A &= \mathrm{NORM}(\mathrm{DIL}(A) \cap \urcorner \mathrm{CON}(A)^2) \\ &= \mathrm{NORM}(\text{more or less but not very very } A) \\ &\doteq 1.234\left(\int_U \mu_A(u)^{0.5} \wedge (1-\mu_A(u)^4)/u\right). \end{aligned} \tag{8}$$

The above are main linguistic hedges proposed by Zadeh. It is found that linguistic hedegs can be viewed as operators which act on a fuzzy set. From this point of view, we can introduce new operators on a fuzzy set. Some of these are given by the following.

The effect of "contrast weakening" (WEAK, for short) is the opposite of that of INT.

$$\mathrm{WEAK}(A) = \int_U \mu_{\mathrm{WEAK}(A)}(u)/u \tag{9}$$

$$\mu_{\mathrm{WEAK}(A)}(u) = \begin{cases} -2(\mu_A(u)^2 - \mu_A(u)) & \cdots\ 0 \leqslant \mu_A(u) \leqslant 0.5 \\ 2(\mu_A(u) - 0.5)^2 + 0.5 & \cdots\ 0.5 \leqslant \mu_A(u) \leqslant 1 \end{cases}$$

The operator of "middle intensification" (MIDI for short) has the effect of intensifing middle grades and is defined as

$$\begin{aligned} \mathrm{MIDI}(A) &= \mathrm{NORM}(A \cap \urcorner A) = 2(A \cap \urcorner A) \\ &= \int_U 2\mu_A(u) \wedge 2(1-\mu_A(u))/u \end{aligned} \tag{10}$$

As the opposite operator to MIDI, we can give MIDW ("middle weakening") as follows.

$$\begin{aligned} \mathrm{MIDW}(A) &= \urcorner\, \mathrm{MIDI}(A) = \urcorner 2(A \cap \urcorner A) \\ &= \int_U (1-2\mu_A(u)) \vee (2\mu_A(u)-1)/u \end{aligned} \tag{11}$$

The operator αCUT obtains the α-level set of a fuzzy set A. The operator α CUT* is the opposite operator to αCUT, that is,

$$\alpha\text{CUT}(A) = \int_U \mu_{\alpha\text{CUT}(A)}(u)/u \tag{12}$$

$$\mu_{\alpha\text{CUT}(A)}(u) = \begin{cases} 1 \cdots \mu_A(u) \geqslant \alpha \\ 0 \cdots \mu_A(u) < \alpha \end{cases}$$

$$\alpha\text{CUT}^*(A) = \int_U \mu_{\alpha\text{CUT}^*(A)}(u)/u \tag{13}$$

$$\mu_{\alpha\text{CUT}^*(A)}(u) = \begin{cases} 1 \cdots \mu_A(u) \leqslant \alpha \\ 0 \cdots \mu_A(u) > \alpha \end{cases}$$

The operator αSCAL which gives the scalor product αA of α and A is defined as

$$\alpha\text{SCAL}(A) = \int_U \alpha\mu_A(u) \wedge 1/u \tag{14}$$

Finally, we shall give two operators which have the effects of "slenderizing" and "swelling" a fuzzy set A. The one is named as αSLND and the other as αSWEL. These have the same expression but they are distinguished from the values of their parameter α. That is to say,

$$\alpha\text{SLND}(A) = \int_U 0 \vee (\alpha\mu_A(u) + 1 - \alpha)/u \cdots \alpha \geqslant 1 \tag{15}$$

$$\alpha\text{SWEL}(A) = \int_U \alpha\mu_A(u) + 1 - \alpha/u \qquad \cdots \alpha \leqslant 1 \tag{16}$$

Fig. 1 shows the effects of the linguistic hedges of (1)—(16) on a fuzzy set A, where A is a fuzzy set $\int_{[0,1]} u/u$ in $U=[0, 1]$.

3. Fuzzy Conditional Inferences

This section discusses a fuzzy conditional inference using the concept of linguistic hedgec. We shall consider the following form of inference in which a fuzzy conditional proposition "If x is A then y is B" is contained.

$$\begin{array}{ll} \text{Prem 1:} & \text{If x is } A \text{ then y is B} \\ \text{Prem 2:} & \text{x is } A' \\ \hline \text{Cons:} & \text{y is } B' \end{array} \tag{17}$$

where x and y are the names of objects, and A, A', B and B' are fuzzy concepts which are represented by fuzzy sets in universes of discourse U, U, V and V, respectively. The form of this inference may be viewed as fuzzy modus ponens which reduces to the classical modus ponens when $A'=A$ and $B'=B$. For simplicity, we shall rewrite (17) as

$$\begin{array}{c} A \Rightarrow B \\ A' \\ \hline B' \end{array} \tag{18}$$

The fuzzy conditional $A \Rightarrow B$ may represent a certain relationship between A and B. From this point of view, Zadeh[2], Mamdani[3], and Mizumoto et al[4-9]. proposed several translating rules for translating $A \Rightarrow B$ into a fuzzy relation in $U \times V$.

Let A and B be fuzzy sets in U and V, respectively, and let $\times$ and $\oplus$ be cartesian product and bounded-sum for fuzzy sets, respectively. Then the following fuzzy relations in $U \times V$ can be translated from $A \Rightarrow B$. The fuzzy relations Rm and Ra were proposed by Zadeh, Rc by Mamdani, and the others by Mizumoto by introducing the implications of many valued logic systems.

$$Rm = (A \times B) \cup (\neg A \times V) = \int_{U \times V} (\mu_A(u) \wedge \mu_B(v)) \vee (1 - \mu_A(u)) / (u, v). \tag{19}$$

$$Ra = (\neg A \times V) \oplus (U \times B) = \int_{U \times V} 1 \wedge (1 - \mu_A(u) + \mu_B(v)) / (u, v). \tag{20}$$

$$Rc = A \times B = \int_{U \times V} \mu_A(u) \wedge \mu_B(v) / (u, v). \tag{21}$$

$$Rs = A \times V \underset{s}{\rightarrow} U \times B = \int_{U \times V} [\mu_A(u) \underset{s}{\rightarrow} \mu_B(v)] / (u, v), \tag{22}$$

where

$$\mu_A(u) \underset{s}{\rightarrow} \mu_B(v) = \begin{cases} 1 & \cdots \ \mu_A(u) \leqslant \mu_B(v), \\ 0 & \cdots \ \mu_A(u) > \mu_B(v). \end{cases}$$

$$Rg = A \times V \underset{g}{\rightarrow} U \times B = \int_{U \times V} [\mu_A(u) \underset{g}{\rightarrow} \mu_B(v)] / (u, v), \tag{23}$$

where

$$\mu_A(u) \underset{g}{\rightarrow} \mu_B(v) = \begin{cases} 1 & \cdots \ \mu_A(u) \leqslant \mu_B(v), \\ \mu_B(v) & \cdots \ \mu_A(u) > \mu_B(v). \end{cases}$$

$$Rb = (\neg A \times V) \cup (U \times B) = \int_{U \times V} (1 - \mu_A(u)) \vee \mu_B(v) / (u, v). \tag{24}$$

$$R_\Delta = A \times V \underset{\Delta}{\rightarrow} U \times B = \int_{U \times V} [\mu_A(u) \underset{\Delta}{\rightarrow} \mu_B(v)] / (u, v), \tag{25}$$

where

$$\mu_A(u) \underset{\Delta}{\rightarrow} \mu_B(v) = \begin{cases} 1 & \cdots \ \mu_A(u) \leqslant \mu_B(v), \\ \dfrac{\mu_B(v)}{\mu_A(u)} & \cdots \ \mu_A(u) > \mu_B(v) \end{cases}$$

In the fuzzy modus ponens of (17) and (18), the consequence B' can be deduced from prem 1 and 2 by taking the max-min compositon "o" of the fuzzy set A' and the fuzzy relation obtained in (19)—(25). For example, we can have

$$Bm' = A' \circ Rm = A' \circ [(A \times B) \cup (\neg A \times V)] \quad (26)$$

The membership function of the fuzzy set Bm' in V is given as

$$\mu_{Bm'}(v) = \bigvee_u \{\mu_{A'}(u) \wedge \mu_{Rm}(u, v)\} \quad (27)$$
$$= \bigvee_u \{\mu_{A'}(u) \wedge [(\mu_A(u) \wedge \mu_B(v)) \vee (1 - \mu_A(u))]\}$$

In the same way, we have

$$Ba' = A' \circ [(\neg A \times V) \oplus (U \times B)] \quad (28)$$

$$Bc' = A' \circ (A \times B) \quad (29)$$

$$\vdots$$

The fuzzy modus ponens of (18) represents that the consequence B' is deduced when the premise A' is given under the condition $A \Rightarrow B$. If we regard the fuzzy conditional $A \Rightarrow B$ (that is, fuzzy relation) as a fuzzy system, A' and B' correspond to "fuzzy input" and "fuzzy output", respectively (see Fig. 2). It will be of interest to discuss what kinds of fuzzy outputs B' are obtaind when various kinds of fuzzy inputs A' are input to a fuzzy system.

In the following we shall obtain B' under each method in (19)-(25) when A' is a fuzzy set given by applying linguistic hedges of (1)-(16) to A, and discuss which method can get reasonable consequences.

we shall discuss only the case of Rm (19) at $A' = A^{\alpha}$ (as a general case of (1)-(5)) because of limitations of space. The similar ways are applicable to the other methods ands A'. Table 1 summarizes the consequences inferrred by all the methods and A'.

When $A' = A^{\alpha}$, the consequence Bm' is obtained from (27) as follows.

$$\mu_{Bm'}(v) = \bigvee_u \{\mu_A(u)^{\alpha} \wedge [(\mu_A(u) \wedge \mu_B(v)) \vee (1 - \mu_A(u))]\} \quad (30)$$

This expression can be rewritten as (32) by letting

$$x = \mu_A(u), \quad b = \mu_B(v), \quad b_{m'} = \mu_{Bm'}(v) \quad (31)$$

under the assumption that $\mu_A(u)$ takes all values in $[0, 1]$ according to u varying all over U, that is, μ_A is a function onto $[0, 1]$, i.e., x is on $[0, 1]$*.

$$b_{m'} = \bigvee_x \{x^{\alpha} \wedge [(x \wedge b) \vee (1 - x)]\} \quad (32)$$

$$f(x) = x^{\alpha} \wedge [(x \wedge b) \vee (1 - x)] \quad (33)$$

Fig. 3 shows the expressions x^{α} and $(x \wedge b) \vee (1 - x)$ using a parameter b. When x^{α} is as in this figure and b is equal to, say, 0.2, the expression $f(x)$ is indicated by the broken line and hence $\bigvee_x f(x)$ at $b = 0.2$ is the maximum value of this broken line. The value is equal to the height of the cross point of x^{α} and $1 - x$. Thus, let x_0 ($\in [0, 1]$) be the solution of $x^{\alpha} = 1 - x$, then the height (i.e., maximum value) is given by $1 - x_0$. Therefore, we have b'_m at $b \leqslant 1 - x_0$ as

$$bm' = \bigvee_x f(x) = 1 - x_0 \cdots b \leqslant 1 - x_0$$

On the other hand, when $b=0.7(\geqslant 1-x_0)$, $f(x)$ is given by the line "——" and its maximum value is b ($=0.7$). Thus

$$bm' = b \cdots b \geqslant 1-x_0$$

Therefore,

$$bm = \begin{cases} 1-x_0 & \cdots\ b \leqslant 1-x_0 \\ b & \cdots\ b \geqslant 1-x_0 \end{cases}$$

$$= (1-x_0) \vee b$$

This result is shown to hold for any α. Therefore, using the notation of (31), the consequence $Bm' = A^\alpha \circ Rm$ is as follows.

$$\mu_{Bm'}(u) = (1-x_0) \vee \mu_B(u) \tag{34}$$

where x_0 ($\in [0,1]$) is the solution of $x^\alpha = 1-x$.

Fig. 4 shows the consequence Bm' ($=A^\alpha \circ Rm$) using a parameter α. Fig. 5 also indicates Ba' ($=A^\alpha \circ Ra$) which can be obtained in the same way as Bm'. Table 1 shows the inference results for other methods and A', where the notation μ_B stands for $\mu_B(v)$.

The consequences B'_Δ for the method $R\Delta(25)$ at $A'=\mathrm{INT}(A)$ and $\mathrm{WEAK}(A)$ are shown in (35) and (36):

Case of $B'_\Delta = \mathrm{INT}(A) \circ R_\Delta$:

$$\mu_{B\Delta'} = \begin{cases} \sqrt[3]{2\mu_B{}^2} & \cdots\ 0 \leqslant \mu_B \leqslant 0.25 \\ \dfrac{\mu_B}{x} & \cdots\ 0.25 \leqslant \mu_B \leqslant 1 \end{cases} \tag{35}$$

where

$$x = \frac{1}{3}\left(2+\sqrt{10}\cos\frac{\ominus+\pi}{3}\right),$$

$$\ominus = \cos^{-1}\left[\frac{\sqrt{10}}{50}(27\mu_B - 14)\right].$$

Case of $B'_\Delta = \mathrm{WEAK}(A) \circ R_\Delta$:

$$\mu_{B\Delta'} = \begin{cases} \dfrac{\mu_B}{x} & \cdots\ 0 \leqslant \mu_B \leqslant 0.25 \\ \dfrac{\mu_B}{x'} & \cdots\ 0.25 \leqslant \mu_B \leqslant 1 \end{cases} \tag{36}$$

where

$$x = \frac{1}{3}\left(1-2\cos\frac{\ominus+\pi}{3}\right)$$

$$\ominus = \cos^{-1}\left(1-\frac{72}{4}\mu_B\right)$$

$$x' = \frac{1}{3}\left\{1+\sqrt[3]{\frac{1}{4}\left(27\mu_B - 5 + 9\sqrt{\frac{27\mu_B{}^2 - 10\mu_B + 1}{3}}\right)} + \sqrt[3]{\frac{1}{4}\left(27\mu_B - 5 - 9\sqrt{\frac{27\mu_B{}^2 - 10\mu_B + 1}{3}}\right)}\right\}$$

The membership functions $\mu_{B\Delta'}$ of (35) and (36) are very complicated and so we shall show in Fig. 6 and 7 the diagrams of $\mu_{B\Delta'}$ together with the results for other methods.

In the form of the fuzzy inference of (18), it is quite natural to expect that $B' = B$ will be obtained when $A' = A$ (the satisfaction of modus ponens). This criterion is satisfied by the methods Rc, Rs and Rg. See the results for A^{α} at $\alpha = 1$ in Table 1. Namely, these methods obtain $A \circ R = B$. Moreover, it is also natural to expect $B' \approx B$ at $A' \approx A$. The method Rs satisfies this criterion and obtains the consequences $B' = B^{\alpha}$, INT(B), WEAK(B), αCUT(B), αSCAL(B), αSLND(B) and αSWEL(B) at $A' = A^{\alpha}$, INT(A), WEAK(A), αCUT(A), αSCAL(A), αSLND(A) and αSWEL(A), respectively. The other methods do not obtain such results. Note that Rc gets always B except the case of αSCAL(A), $\alpha < 1$. In[4] we showd that all the methods except Rc satisfy the natural criterion that B' = unknown is obtained at A' = not A, where unknown is defined as $\int_V \frac{1}{V}$. In this connection, for the criterion that $B' \approx$ unknown is obtained at $A' \approx$ not A, we may say that all the methods except Rc satisfy this criterion. In fact, these methods get B' = unknown at A' = MIDW(A) and αCUT*(A) which are similar to not A. Finally, it is not yet known what kinds of consequences are good for A' = sort of A, slightly A and MIDI(A) which are between A and not A.

From the above considerations, we can conclude that the method Rs (22) is most suitable for the fuzzy conditional inference, though the given criteria are intuitive and rough.

4. Conclusion

Under the criterion that $B' \approx B$ at $A' \approx A$ and $B' \approx$ unknown at $A' \approx$ not A for the fuzzy reasoning of (18), it will be possible to make a quantitative analysis of the goodness of each method by measuring a similarity of B' and B (or unknown) when A' is given which is similar to A (or not A).

The results of this paper will be useful to the problems such as fuzzy control, fuzzy diagnosis, fuzzy production system and so on which use fuzzy reasoning methods.

References

[1] Zadeh, L. A., A fuzzy-set-theoretic interpretation of linguistic hedges, J. of Cybernetics, 2, 4, 4-34, 1972.

[2] Zadeh, L. A., Calculus of fuzzy restriction, in Fuzzy Sets and Their Applications to Cognitive and Decision Processes, (ed. by Zadeh, L. A. et al.) Acad. Press, pp. 1-39, 1975.

*The case where μ_A is not a function onto[0,1], say, μ_A is discrete, will be discussed in the subsequent papers.

[3] Mamdani, E.H., Application of fuzzy logic to approximate reasoning using linguistic systems, IEEE Trans. on Comp., c-26, 1182-1191, 1977.

[4] Mizumoto, M. & Zimmermann, H. J., Comparison of fuzzy reasoning methods, Fuzzy Sets and Systems (in Press).

[5] Fukami, S., Mizumoto, M. & Tanaka, K., Some considerations on fuzzy conditional inference, Fuzzy Sets and Systems, 4, 243-273, 1980.

[6] Mizumoto, M., Fukami, S. & Tanaka, K., Several methods for fuzzy conditional inference, Proc. IEEE Conf. on Decision and Control (Florida, Dec. 12-14), 777-782, 1979.

[7] Mizumoto, M., Fukami, S. & Tanaka, K., Some methods of fuzzy reasoning, in Advances in Fuzzy Set Theory and Applications (ed. Gupta, M. M. et al.), North-Holland, 117-136, 1979.

[8] Mizumoto, M., Fukami, S. & Tanaka, K., Fuzzy conditional inference and fuzzy inference with fuzzy quantifiers, Proc. 6th Int. Conf. on Artificial Intelligence (Tokyo, Aug. 20-23), 589-591, 1979.

[9] Mizumoto, M., Fukami, S. & Tanaka, K., Fuzzy reasoning methods by Zadeh and Mamdani, and improved methods, 3rd Workshop on Fuzzy Reasoning, London, Sept. 15, 1978.

Table 1 Inference results under each method

	R_m	R_a	R_c	R_s	R_g	R_b	▲
A^α	$(1-x_0)\vee\mu_B$ (*)	$1-x'+\mu_B$ (**)	μ_B	μ_B^α	$\begin{cases}\mu_B^\alpha & \cdots\ \alpha\le 1\\ \mu_B & \cdots\ \alpha\ge 1\end{cases}$	$(1-x_0)\vee\mu_B$ (*)	μ_B^{α}
INT(A)	$0.5\vee\mu_B$	$\frac{4\mu_B-1+\sqrt{9-8\mu_B}}{4}$	μ_B	$\begin{cases}2\mu_B^2 & \cdots\ 0\le\mu_B\le 0.5\\ 1-2(1-\mu_B)^2 & \cdots\ 0.5\le\mu_B\le 1\end{cases}$	$\begin{cases}\mu_B & \cdots\ 0\le\mu_B\le 0.5\\ 1-2(1-\mu_B)^2 & \cdots\ 0.5\le\mu_B\le 1\end{cases}$	$0.5\vee\mu_B$	See (35
slightly A	$\left(\frac{\sqrt{5}-1}{2}\vee\mu_B\right)\wedge m_0$ (***)	$\frac{\sqrt{5}-1}{2}(1+\mu_B)\wedge 1$	$\mu_B\wedge m_0$	$\frac{\sqrt{5}+1}{2}\mu_B\wedge 1$	$\frac{\sqrt{5}+1}{2}\mu_B\wedge 1$	$\frac{\sqrt{5}-1}{2}\vee\mu_B$	$\sqrt{\frac{\sqrt{5}+1}{2}\mu_B}\wedge 1$
sort of A	$(0.689\vee\mu_B)\wedge 0.779$	$\frac{c(\sqrt{4(1+\mu_B)+c^2}-c)}{2}\wedge 1$ $c=1.234\cdots$	$\mu_B\wedge 0.779$	$c\sqrt{\mu_B}\wedge 1$ (****)	$c\sqrt{\mu_B}\wedge 1$	$0.689\vee\mu_B$	$\sqrt[3]{c^2\mu_B}\wedge 1$
WEAK(A)	$0.5\vee\mu_B$	$\frac{3+4\mu_B-\sqrt{1+8\mu_B}}{4}$	μ_B	$\begin{cases}-2(\mu_B^2-\mu_B) & \cdots\ 0\le\mu_B\le 0.5\\ 2(\mu_B-\frac{1}{2})^2+\frac{1}{2} & \cdots\ 0.5\le\mu_B\le 1\end{cases}$	$\begin{cases}-2(\mu_B^2-\mu_B) & \cdots\ 0\le\mu_B\le 0.5\\ \mu_B & \cdots\ 0.5\le\mu_B\le 1\end{cases}$	$0.5\vee\mu_B$	See (36)
MIDI(A)	$\frac{2}{3}$	$\frac{2}{3}(1+\mu_B)\wedge 1$	$\mu_B\wedge\frac{2}{3}$	$2\mu_B\wedge 1$	$2\mu_B\wedge 1$	$\frac{2}{3}\vee\mu_B$	$\sqrt{2\mu_B}$ 1
MIDW(A)	1	1	$\frac{1}{3}\vee\mu_B$	1	1	1	1
αCUT(A)	$(1-\alpha)\vee\mu_B$	$(1-\alpha+\mu_B)\wedge 1$	μ_B	$\begin{cases}1 & \mu_B\ge\alpha\\ 0 & \mu_B<\alpha\end{cases}$	$\begin{cases}1 & \cdots\ \mu_B\ge\alpha\\ \mu_B & \cdots\ \mu_B<\alpha\end{cases}$	$(1-\alpha)\vee\mu_B$	$\frac{\mu_B}{\alpha}\wedge 1$
αCUT*(A)	1	1	$\mu_B\wedge\alpha$	1	1	1	1
αSCAL(A)	$\begin{cases}(\frac{\alpha}{\alpha+1}\vee\mu_B)\wedge\alpha & \cdots\ \alpha\le 1\\ \frac{\alpha}{\alpha+1}\vee\mu_B & \alpha\ge 1\end{cases}$	$\begin{cases}\frac{\alpha}{\alpha+1}(1+\mu_B)\wedge\alpha & \cdots\ \alpha\le 1\\ \frac{\alpha}{\alpha+1}(1+\mu_B)\wedge 1 & \cdots\ \alpha\ge 1\end{cases}$	$\begin{cases}\mu_B\wedge\alpha & \cdots\ \alpha\le 1\\ \mu_B & \alpha\ge 1\end{cases}$	$\begin{cases}\alpha\mu_B & \alpha\le 1\\ \alpha\mu_B\wedge 1 & \alpha\ge 1\end{cases}$	$\begin{cases}\mu_B\wedge\alpha & \cdots\ \alpha\le 1\\ \alpha\mu_B\wedge 1 & \cdots\ \alpha\ge 1\end{cases}$	$\begin{cases}(\frac{\alpha}{\alpha+1}\vee\mu_B)\wedge\alpha & \cdots\ \alpha\le 1\\ \frac{\alpha}{\alpha+1}\vee\mu_B & \cdots\ \alpha\ge 1\end{cases}$	$\begin{cases}\sqrt{\alpha\mu_B}\wedge\alpha & \alpha\le 1\\ \sqrt{\alpha\mu_B}\wedge 1 & \alpha\ge 1\end{cases}$
αSLND(A) ($\alpha\ge 1$) / αSWEL(A) ($\alpha\le 1$)	$\frac{1}{1+\alpha}\vee\mu_B$	$\frac{1+\alpha\mu_B}{1+\alpha}$	μ_B	$0\vee(\alpha\mu_B+1-\alpha)$	$\frac{\mu_B}{\alpha\mu_B+1-\alpha}$	$\frac{1}{1+\alpha}\vee\mu_B$	$\frac{1-\alpha+\sqrt{(1-\alpha)^2+4\alpha\mu_B}}{2}$

(*) x_0 ($\in[0,1]$) is the solution of $x^\alpha=1-x$ (**) x ($\in[0,1]$) is the solution of $x^\alpha=1-x+\mu_B$ (***) $m_0=\frac{1-\sqrt{5}+\sqrt{22-2\sqrt{5}}}{4}\doteq 0.7376\ldots$ (****) $c=1.234\ldots$

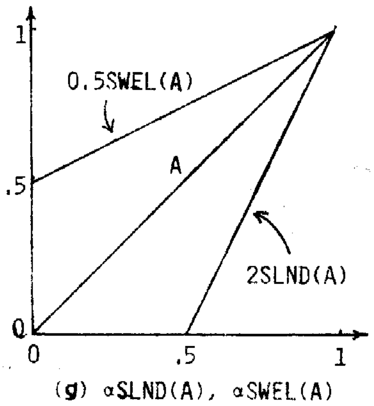


Fig. 1 (continued)

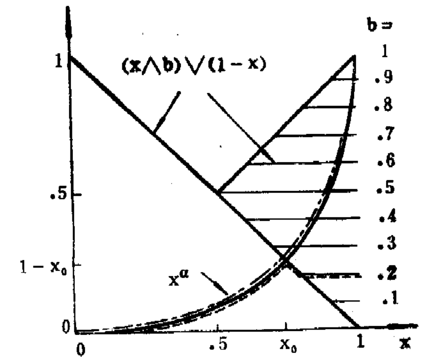


Fig. 3 The way of obtaining (32)

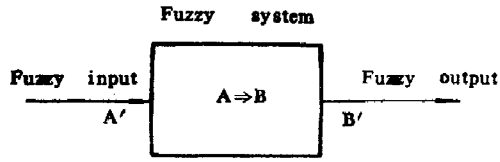


Fig. 2 Fuzzy system (A⇒B) with fuzzy input A' and fuzzy output B'

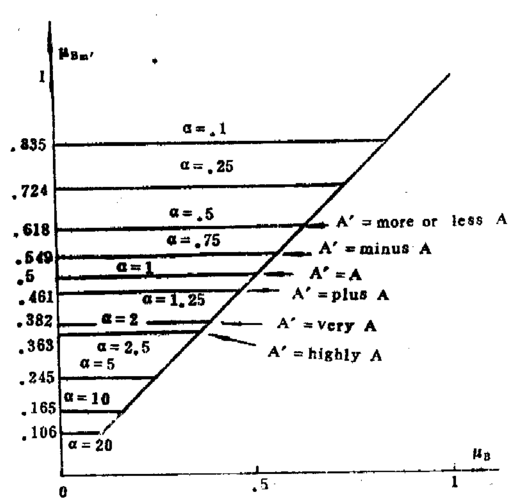


Fig. 4 $B_{m'} = A'^{\bullet}R_m$

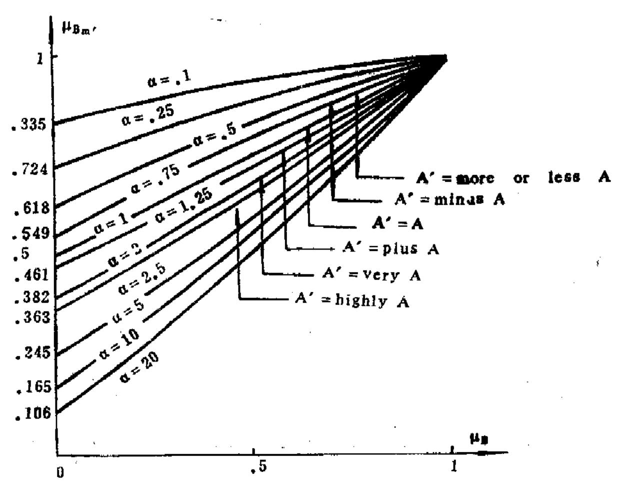


Fig. 5 $B_{a'} = A'^{\bullet}R_a$

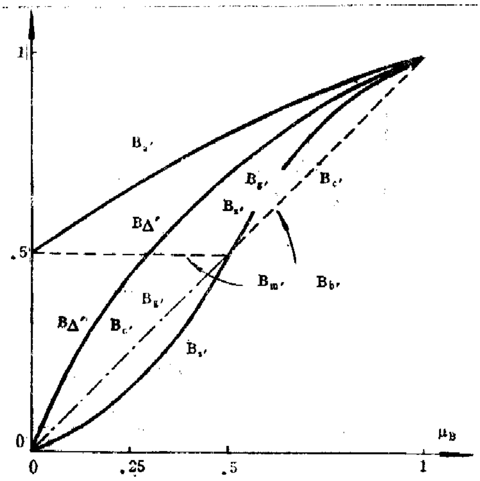


Fig. 6 Inference results at A' = INT(A)

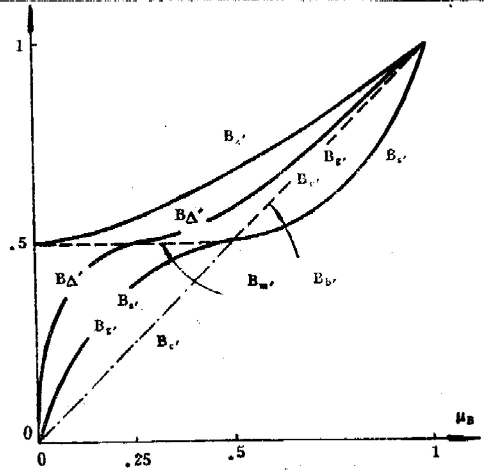
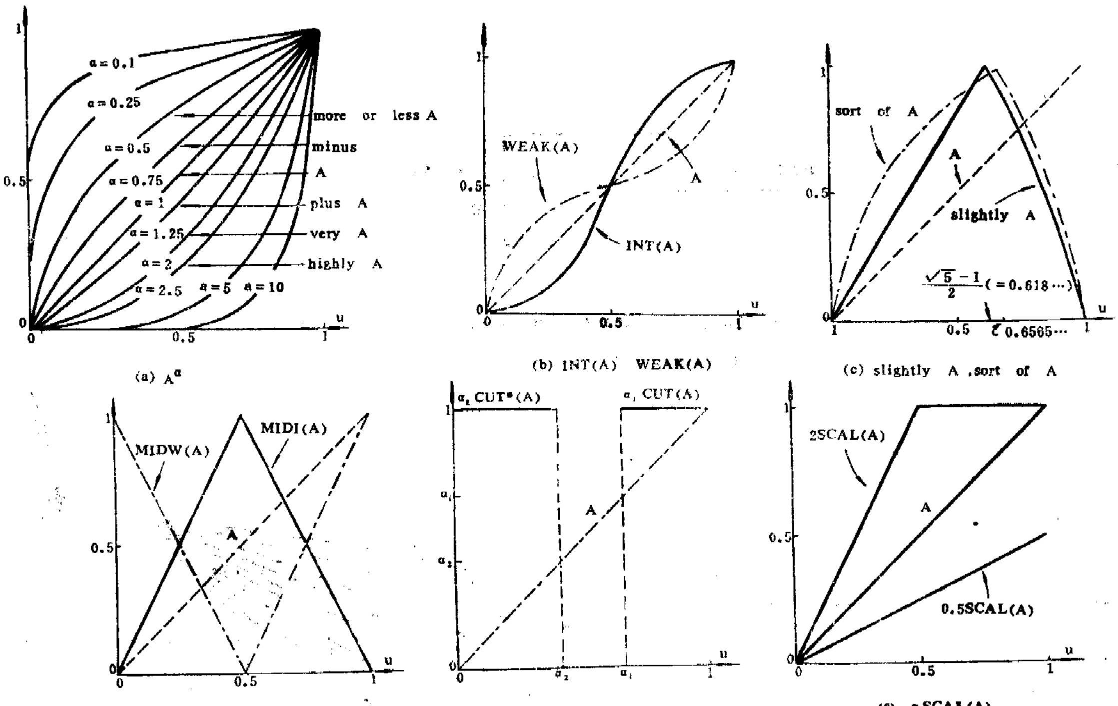


Fig. 7 Inference results at A' = WEAK(A)



(a) A^α (b) INT(A) WEAK(A) (c) slightly A, sort of A

(d) MIDI(A) MIDW(A) (e) α CUT(A), α CUT*(A) (f) α SCAL(A)

ig. 1 Various linguistic hedges for a fuzzy set A

各种不同Fuzzy输入的Fuzzy推理

M. Mizumoto

（日本 大阪电信大学）

本文比较了几种不同翻译规则下Fuzzy条件推理

$$\frac{\begin{array}{c}A\Rightarrow B\\ A'\end{array}}{B'}\quad (A, A', B, B'\text{：Fuzzy集})$$

的推理结果，这里前提A'是在A上施加诸如“稍微”、“之类”、“最”等限制词得到的Fuzzy集合。本文证明作者提出的翻译规则Rs可以得到与直观相符的相当合理的推理结果。本文的结果对Fuzzy控制、Fuzzy诊断及Fuzzy生产过程都是有用的。