A SYSTEM FOR FUZZY REASONING

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This paper describes an implementation of a system for fuzzy reasoning which facilitates the execution of approximate reasoning from the specified propositions and gives the result in both fuzzy-set and linguistic forms. The description of the Approximate Reasoning System contains the Universe of Discourse Block which defines several universes of discourse; the Primary Term Block which defines several primary terms; the Proposition Block which states propositions using primary terms, linguistic hedges and IF ... THEN ...; and the Approximate Reasoning Block which executes approximate reasoning and linguistic approximation. The Approximate Reasoning System is implemented using the FSTDS System for fuzzy-set manipulation and is running on a FACOM 230-45S computer.

1. INTRODUCTION

Much human reasoning is fuzzy rather than precise in nature. An example of such reasoning may be "If x is *small* and x and y are *approximately equal*, then y is *more or less small*". The fuzziness in the words: *small*, *approximately equal* etc. may be defined by fuzzy sets and fuzzy relations[1]. Reasoning with fuzzy concepts has been formulated by Zadeh[2] who calls it approximate reasoning.

In this paper, we describe a system for fuzzy reasoning, called an Approximate Reasoning System, which is based on Zadeh's formulation of fuzzy reasoning. This system facilitates the definitions of universes of discourse and primary terms, the description of fuzzy propositions, and the execution of approximate reasoning using the specified fuzzy propositions. The result of approximate reasoning is given in both fuzzy-set and linguistic forms.

2. APPROXIMATE REASONING AND LINGUISTIC HEDGES

Zadeh's formulation of approximate reasoning is defined as the compositional rule of inference which is expressed in symbols[2] as

$$\begin{array}{l} P_1\text{: x is A.} \\ P_2\text{: x and y are R.} \\ \hline P_3\text{: y is A}\circ\text{R.} \end{array} \qquad (1)$$

where x and y are object names, A is a fuzzy set in U, R is a fuzzy relation in U×V, and A∘R is the composition of A and R, i.e., is a fuzzy set in V. If P_2 is a conditional proposition such as

$$P_2\text{: If x is P then y is Q.} \qquad (2)$$

where x and y are object names and P and Q are fuzzy sets in U and V, respectively, then it is translated into the fuzzy relation R of x and y using the arithmetic rule for conditional propositions[2],

$$R = (\neg P \times V) \oplus (U \times Q) \qquad (3)$$

where ×, ¬ and ⊕ stand for the Cartesian product, the complement and the bounded sum, respectively.

[Example 1] Let U be a universe of discourse expressed by

$$U = \{1, 2, 3, 4\}.$$

If we have the fuzzy set *small* in U as

$$small = \{1/1, 0.6/2, 0.2/3\}$$

and the fuzzy relation *approximately equal* in U×U as

$$\begin{aligned} approximately\ equal = \{&1/\langle1,1\rangle, 1/\langle2,2\rangle, 1/\langle3,3\rangle, 1/\langle4,4\rangle, \\ &0.5/\langle1,2\rangle, 0.5/\langle2,3\rangle, 0.5/\langle3,4\rangle\}, \end{aligned}$$

then from the premise propositions:

P_1: x is *small.*
P_2: x and y are *approximately equal.*

we can infer by approximate reasoning the consequence:

$$y = small \circ approximately\ equal = \{1/1,\ 0.6/2,\ 0.5/3,\ 0.2/4\}.$$

A linguistic hedge such as *very*, *more or less*, *much* and *slightly* can be viewed as an operator which operates on the operand fuzzy set. For example, several hedges are defined by Zadeh[3] and Lakoff[4].

The effects of some linguistic hedges are shown in Fig.1 in the case where a fuzzy set is *tall*[4].

We can obtain a fuzzy set as the consequence of approximate reasoning. If the consequence remains in fuzzy-set form, it is difficult for us to understand its meaning, as was the case in Example 1. So the consequence would be better given "*more or less small*" rather than a fuzzy set {1/1, 0.6/2, 0.5/3, 0.2/4}. Thus representing a fuzzy set approximately in a linguistic form by some appropriate hedges and fuzzy sets already defined is called linguistic approximation.

3. APPROXIMATE REASONING SYSTEM

The Approximate Reasoning System (AR System for short) facilitates the execution of approximate reasoning from the specified propositions and outputs the consequence in a linguistic form by linguistic approximation.

We shall illustrate the facilities of the Approximate Reasoning System using a simple example.

[Example 2] If we have a description in the AR System as in Fig.2, then we obtain the output as in Fig.3.

The UNIVERSE OF DISCOURSE BLOCK defines a universe of discourse U and the PRIMARY TERM BLOCK defines fuzzy sets SMALL, MIDDLE and LARGE on U. In the PROPOSITION BLOCK, we describe the propositions P1 and P2. In the AR BLOCK (APPROXIMATE REASONING BLOCK), PARA sets parameters (the threshold value of the evaluation function and the depth of nested hedges) for linguistic approximation and AR executes

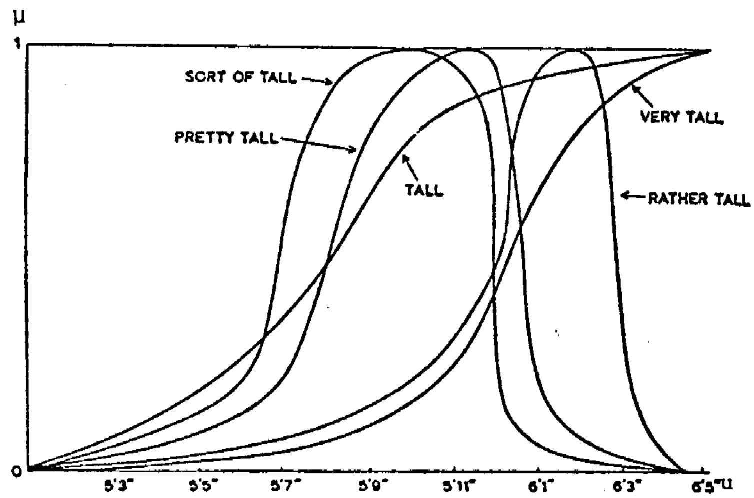

Fig.1. The effect of hedges *very*, *sort of*, *rather* and *pretty* (Lakoff[4]).

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******* UNIVERSE OF DISCOURSE BLOCK *********
    U=(1, 2, 3, 4, 5);
******* PRIMARY TERM BLOCK *****************
/U/ SMALL=(1/1, 0.8/2, 0.5/3, 0.2/4);
/U/ MIDDLE=(0.2/1, 0.5/2, 1/3, 0.5/4, 0.2/5);
/U/ LARGE=(0.2/2, 0.5/3, 0.8/4, 1/5);
******* PROPOSITION BLOCK ******************
P1: X IS LARGE;
P2: IF X IS VERY LARGE THEN Y IS NOT LARGE;
******* AR BLOCK ***************************
    PARA(C=2,T=13);
    AR(P1,P2/SMALL MIDDLE LARGE);
******* END OF APPROXIMATE REASONING ********
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Fig.2. A description in the Approximate Reasoning System.

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..... OBTAINED SUFFICIENT LINGUISTIC APPROXIMATION

* LINGUISTIC APPROXIMATION...  MINUS MORE OR LESS SMALL

* PRIMARY TERM...  SMALL

* FUZZY SET WHICH IS PRIMARY TERM...
    FSET(1/1, 0.8/2, 0.5/3, 0.2/4);

* FUZZY SET GENERATED BY COMPOSITIONAL RULE OF INFERENCE...
    FSET(1/1, 0.96/2, 0.8/3, 0.56/4, 0.5/5);

* FUZZY SET MODIFIED BY SOME HEDGES...
    FSET(1/1, 0.9197/2, 0.771/3, 0.5468/4);

* EVALUATION FUNCTION VALUE...  0.1165

* THRESHOLD VALUE...  0.130
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Fig.3. A result of the Approximate Reasoning System.

approximate reasoning from the propositions P1 and P2, and approximates the consequence linguistically by the primary terms SMALL, MIDDLE and LARGE.

As shown in Example 2, a description in the AR System contains four blocks, namely, the Universe of Discourse Block, Primary Term Block, Proposition Block and Approximate Reasoning (AR) Block. We have no more detailed explanation of their blocks on account of limited space.

4. IMPLEMENTATION OF THE AR SYSTEM

The Approximate Reasoning System is implemented using the FSTDS System (Fuzzy-Set-Theoretic Data Structure System)[5] which enables a user to write a program in FSTDSL/FORTRAN using 52 fuzzy-set operators on fuzzy sets and fuzzy relations.

The AR System passes to the FSTDS System the sets and fuzzy sets defined in the Universe of Discourse Block and Primary Term Block. A composite term is translated into an expression in FSTDSL, that is, hedges are replaced by the corresponding operators in FSTDSL and connectives are translated into prefix operators. A conditional proposition IF ... THEN ... is translated by (3).

For the AR statement, the AR System passes the statement:

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%CONSEQUENCE = IMAGE(P2, P1);
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to the FSTDS System. Note that AoR in (2) reduces to the image of A under R since A is a unary fuzzy relation.

For the linguistic approximation, we can not use the really best linguistic approximation, since it generates too many composite terms. Therefore we use the following strategy.

First, we apply each hedge to given m primary terms and compare its result with the consequence fuzzy set B. If we obtain a sufficient approximation, that is, the value of evaluation function is less than or equal to the threshold value, then it is accepted as the linguistic approximation of B. Otherwise we choose one composite term of the best approximation in the first step of linguistic approximation and apply each hedge again to it. If we cannot still obtain a good one, the same is repeated n times. If a sufficient approximation has not been obtained yet after n repetitions, the best approximation by this time would be considered as a linguistic approximation of B. There is, however, no guarantee for the really best approximation.

For the evaluation function, there might be several definitions. We have adopted the evaluation function such as

$$f(B,B') = \frac{\sum |\mu_B(u) - \mu_{B'}(u)|}{\#(U)} \tag{4}$$

where B is the fuzzy set obtained by approximate reasoning, B' the fuzzy set modified by hedges in linguistic approximation and U a universe of discourse of B and B', and #(U) denotes the number of elements in U, $|x|$ the absolute value of real number x and Σ the ordinary sum over U.

5. CONCLUSION

We have described an Approximate Reasoning System which facilitates the definition of universes of discourse and primary terms, the description of propositions and execution of approximate reasoning and linguistic approximation. In the implementation of the AR System, the translation of the description in the AR System to FSTDSL is programmed in PL/I and the construction and manipulation of fuzzy sets and fuzzy relations are programmed in FSTDSL/FORTRAN[5], and it is currently running on a FACOM 230-45S computer. The current AR System is an interpreter version.

As a basis of question/answering system and intelligent access to database systems, fuzzy reasoning plays so important role in the interface between such systems and users that we expect the AR System will find various other applications.

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