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Implementation of Approximate Reasoning System Using FSTDS System

by

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Abstract

This paper describes the implementation of Approximate Reasoning System which facilitates the execution of approximate reasoning from the specified propositions and gives the result in both fuzzy-set and linguistic forms. The description of Approximate Reasoning System contains the Universe of Discourse Block which defines several universes of discourse; Primary Term Block which defines several primary terms; Proposition Block which states propositions using primary terms; and Approximate Reasoning Block which executes approximate reasoning and linguistic approximation.

Approximate Reasoning System is implemented using FSTDS system and is running on a FACOM 230-45S computer.

1. Introduction

Much of human reasoning is approximate rather than precise in nature. An example of such reasoning may be "x is *small* and x and y are *approximately equal*, then y is *more or less small*." The fuzziness in the words: *small*, *approximately equal* etc. may be defined by fuzzy sets and fuzzy relations[1]. The reasoning with fuzzy concepts has been formulated by Zadeh[2,3] who calls it approximate reasoning.

In this paper, we describe Approximate Reasoning System which is baed on Zadeh's formulation of approximate reasoning. This system facilitates the definitions of universes of discourse on which primary terms are defined, the description of fuzzy propositions and the execution of approximate reasoning using the specified fuzzy propositions. The result of approximate reasoning is given in both fuzzy-set and linguistic forms.

Approximate Reasoning System is implemented using FSTDS System (Fuzzy-Set-Theoretic Data Structure System)[4] which enables us to write a program in FSTDSL/FORTRAN using 52 fuzzy-set operators on fuzzy sets and fuzzy relations, and is currently running on a FACOM 230-45S computer.

In Section 2 we outline Zadeh's formulation of approximate reasoning and the concept of a linguistic hedge. We then describe Approximate Reasoning System by illustrating several examples in Section 3. The section contains also the method of linguistic approximation in our system. The final section discusses the limitation and future extension of our system.

2. Approximate Reasoning and Linguistic Hedges

Zadeh's formulation of approximate reasoning is defined as the compositional rule of inference which is expressed in symbols as

$$\frac{\begin{array}{l} P_1\text{: x is A.} \\ P_2\text{: x and y are R.} \end{array}}{P_3\text{: y is } A \circ R.} \qquad (2.1)$$

where x and y are object names, A is a fuzzy set in U, R is a fuzzy relation in U×V, and A∘R is the composition of A and R, i.e., is a fuzzy set in V.

If P_2 is a conditional proposition such as

$$P_2\text{: If x is P then y is Q.} \qquad (2.2)$$

where x and y are object names and P and Q are fuzzy sets in U and V, respectively, then it is translated into the fuzzy relation R of x and y using either the maximin rule for conditional propositions [5],

$$R = P \times Q \cup \sim P \times V \qquad (2.3)$$

or the arithmetic rule for conditional propositions [5],

$$R = \sim P \times V \oplus U \times Q \qquad (2.4)$$

where ×, ∪, ∼ and ⊕ stand for the Cartesian product, the union, the complement and the bounded sum, respectively.

[Example 1] Let U be a universe of discourse expressed by

$$U = \{1, 2, 3, 4\}.$$

If we have a fuzzy set *small* in U as

$$small = \{1/1, 0.6/2, 0.2/3\}$$

and a fuzzy relation *approximately equal* in U×U as

$$approximately\ equal = \sum_{U \times U} \max(0, 1 - \frac{|u_1 - u_2|}{2}) / \langle u_1, u_2 \rangle,$$

then from the premise propositions:

P_1: x is *small*.

P_2: x and y are *approximately equal*.

we can infer by approximate reasoning the consequence:

$$\begin{aligned} y &= small \circ approximately\ equal \\ &= \{1/1, 0.6/2, 0.5/3, 0.2/4\}. \end{aligned}$$

[Note] We did not use in our system the formulation of the conditional proposition such as

If x is P then y is Q else y is S. (2.5)

since it might have two meanings in natural languages. For example, consider the proposition:

If x is *small* then y is Q else y is S. (2.6)

The proposition (2.6) might have two meanings. The one might be

If x is *small* then y is Q and if x is *not small* then y is S. (2.7)

and the other might be

If x is *small* then y is Q and if x is *large* then y is S. (2.8)

The definition of "*not small*" is not in general the same as that of "*large*". This is the reason why we omit the form of proposition (2.5) in our system. Thus we adopt the explicit form of proposition (2.7) or (2.8) rather than (2.6).

A linguistic hedge such as *very*, *more or less*, *much* and *slightly* can be viewed as an operator which operates on the operand fuzzy set. For example, several hedges are defined by Zadeh [6] and Lakoff [7] as follows:

$$\textit{very}\ A = \mathrm{CON}(A) = A^2 \tag{2.9}$$

$$\textit{more or less}\ A = \mathrm{DIL}(A) = A^{0.5} \tag{2.10}$$

$$\textit{plus}\ A = A^{1.25} \tag{2.11}$$

$$\textit{minus}\ A = A^{0.75} \tag{2.12}$$

$$\begin{aligned}\textit{slightly}\ A &= \mathrm{NORM}(A \cap \sim\mathrm{CON}(A)) \\ &\text{or } \mathrm{CINT}(\mathrm{NORM}(\textit{plus}\ A \cap \sim\mathrm{CON}(A))) \\ &\text{or } \mathrm{CINT}(\mathrm{NORM}(\textit{plus}\ A \cap \sim\textit{plus}\ \mathrm{CON}(A)))\end{aligned} \tag{2.13}$$

$$\textit{sort of}\ A = \mathrm{NORM}(\sim\mathrm{CON}(\mathrm{CON}(A)) \cap \mathrm{DIL}(A)) \tag{2.14}$$

$$\begin{aligned}\textit{rather}\ A &= \mathrm{CINT}(\mathrm{CON}(A)) \\ &\text{or } \mathrm{CINT}(\mathrm{CON}(A)) \cap \sim\mathrm{CON}(A)\end{aligned} \tag{2.15}$$

$$\textit{pretty}\ A = \mathrm{CINT}(A) \cap \sim\mathrm{CINT}(\mathrm{CON}(A)) \tag{2.16}$$

where A is a fuzzy set, A^r represents $\Sigma(\mu_A(u))^r/u$ (r: real number), $\sim$ and $\cap$ mean the complement and the intersection, respectively, and CON, DIL, NORM and CINT stand for the concentration, the dilation, the normalization and the contrast intensification, respectively. The artificial hedges *plus* and *minus* are introduced to provide milder degrees of concentration and dilation than those associated with CON and DIL [6].

The effects of some of the above linguistic hedges are shown in Fig.1 in the case where a fuzzy set A is *tall* [7].

We can obtain a fuzzy set as the consequence of approximate reasoning. If the consequence remains in the fuzzy-set form, it is difficult for us to understand the meaning of the consequence, as was the case in Example 1. So the consequence fuzzy set had better to be representd in the linguistic form. In Example 1, if y would be given *more or less small* rather than a fuzzy set {1/1, 0.6/2, 0.5/3, 0.2/4}, we could obtain the consequence "y is *more or less small*.", which might be much easier to understand its meaning.

Thus representing a fuzzy set approximately in a linguistic form by some appropriate hedges and fuzzy sets already defined is called linguistic approximation.

3. Approximate Reasoning System

Approximate Reasoning System (AR System for short) facilitates the execution of approximate reasoning from the specified fuzzy propositions and outputs

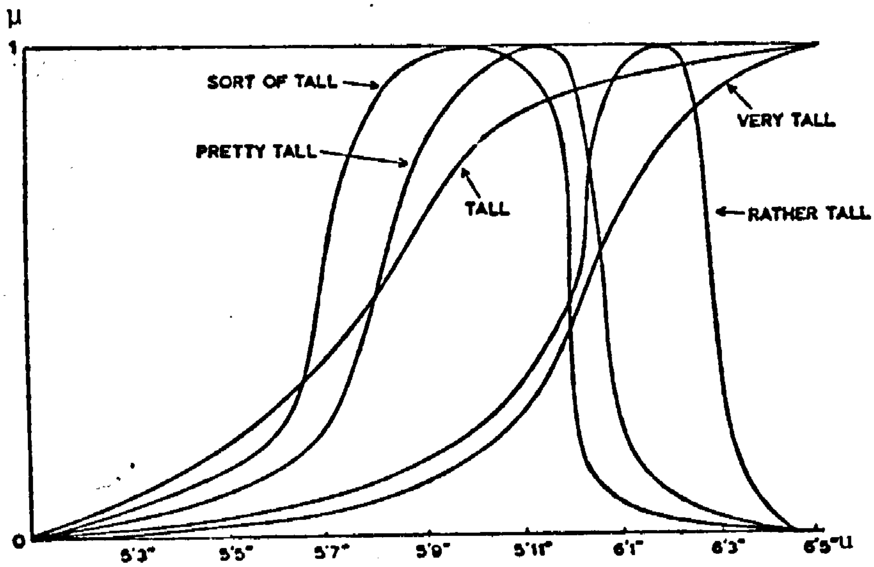

Fig.1. The effects of hedges *very*, *sort of*, *rather* and *pretty* (cf. Lakoff [7]).

the consequence in a linguistic form by linguistic approximation.

First of all, we will give the facilities of Approximate Reasoning System by a simple example.

[Example 2] If we have the description in AR System as follows:

```
********* UNIVERSE OF DISCOURSE BLOCK **********
      U=(1, 2, 3, 4, 5);
********* PRIMARY TFRM BLOCK ******************
  /U/ SMALL=(1/1, 0.8/2, 0.5/3, 0.2/4);
  /U/ MIDDLE=(0.2/1, 0.5/2, 1/3, 0.5/4, 0.2/5);
  /U/ LARGE=(0.2/2, 0.5/3, 0.8/4, 1/5);
********* PROPOSITION BLOCK *******************
  P1: X IS LARGF;
  P2: IF X IS VERY LARGE THEN Y IS NOT LARGE;
********* AR BLOCK ****************************
      PARA(C=2,T=12);
      AR(P1,P2/SMALL MIDDLE LARGE);
********* END OF APPROXIMATE REASONING *********
```

then the consequence in a fuzzy-set form:

```
FSET(1/1, 0.96/2, 0.8/3, 0.56/4, 0.5/5);
```

and its linguistic approximation:

```
MINUS MORE OR LESS SMALL
```

are obtained. The UNIVERSE OF DISCOURSE BLOCK defines a universe of discourse U and the PRIMARY TERM BLOCK defines fuzzy sets SMALL, MIDDLE and LARGE on U. In the PROPOSITION BLOCK, we describe the propositions P1 and P2. In the AR BLOCK (APPROXIMATE REASONING BLOCK), PARA sets parameters for linguistic approximation and AR executes approximate reasoning from the premise propositions P1 and P2, and approximates the consequence linguistically by the primary terms SMALL, MIDDLE and LARGE.

More detailed information is output by AR System, but we have omitted it for simplicity. Detailed results from the system will appear in the later example.

As was shown in Example 2, the description in AR System contains four blocks, namely, the Universe Discourse Block, Primary Term Block, Proposition Block and Approximate Reasoning (AR) Block.

(1) Universe of Discourse Block

This block defines several universes of discourse on which fuzzy sets and fuzzy relations in the Primary Term Block are defined.

The definition of a universe of discourse is written in general as

<universe of discourse name> = <definition part>; (3.1)

where <universe of discourse name> is a string of characters and <definition part> can be an expression in FSTDSL [4] except that a set-construction operator SET may be omitted, e.g., (1,2,3) is equivalent to SET(1,2,3).

(2) Primary Term Block

The Primary Term Block defines several primary terms which will appear in the definition of propositions in the Proposition Block.

A user has to put the universe of discourse name at the head of each definition of primary term to specify which fuzzy set of primary term is defined on which universe of discourse. Thus, a general form of definition of a primary term is

/<universe of discourse name>/ <primary term name> = <definition part>; (3.2)

where <universe of discourse name> must be defined in the Universe of Discourse Block, <primary term name> is a character string and <definition part> is again an expression in FSTDSL except that in this case a fuzzy-set-construction operator FSET can be omitted.

(3) Proposition Block

This block describes several propositions which are used as the premises in the Approximate Reasoning Block. We can describe an object or a relationship of objects using primary terms, IF ... THEN ... and composite terms which are combinations of primary terms, hedges and connectives [2].

The definition of proposition is as follows:

<proposition name> : <proposition part>; (3.3)

where <proposition name> is a string of characters and <proposition part> is defined by either

$$\langle\text{object name}\rangle\ [\text{AND}\ \langle\text{object name}\rangle]\ \left\{\begin{matrix}\text{IS}\\ \text{ARE}\end{matrix}\right\}\ \left\{\begin{matrix}\langle\text{primary term name}\rangle\\ \langle\text{composite term}\rangle\end{matrix}\right\} \qquad (3.4)$$

or

$$\text{IF}\ \langle\text{proposition}\rangle\ \text{THEN}\ \langle\text{proposition}\rangle$$
$$\left[\ \left\{\begin{matrix}\text{AND}\\ \text{OR}\end{matrix}\right\}\ \text{IF}\ \langle\text{proposition}\rangle\ \text{THEN}\ \langle\text{proposition}\rangle\ \right] \qquad (3.5)$$

where [...] means arbitrary times of repetitions of the elements and {...} a selection of one element, and <object name> is a character string. <composite term> is generated by a finite times of applications of hedges and connectives to primary terms. The formal definition of <composite term> was omitted on account of limited space. Currently, nine hedges, namely, *very*, *more or less*, *plus*, *minus*, *slightly*, *sort of*, *rather*, *pretty* and *not* and two connectives *and* and *or* are available in the Proposition Block. The hedge *not* and connectives *and* and *or* are interpreted as the complement, the intersection and the union, respectively. The other hedges are interpreted as (2.9)-(2.16).

(4) Approximate Reasoning Block

This block has two instructions AR and PARA. The AR instruction specifies the propositions from which approximate reasoning is executed, and the list of primary terms which are used for linguistic approximation of the consequence.

The AR instruction has the form in general:

AR(<proposition name>,<proposition name>/<primary term list>); (3.6)

where <proposition name> must be defined in the Proposition Block and <primary term list> is an arbitrary number of primary terms separated by space symbols.

Thus the instruction in Example 2:

```
AR(P1,P2/SMALL MIDDLE LARGE);
```

represents the execution of approximate reasoning from the propositions P1 and P2 and the linguistic approximation of the consequence by the primary terms SMALL, MIDDLE and LARGE. Note that only hedges are used but connectives are not used in linguistic approximation because of human's understandability.

By PARA instruction we can specify the conditions in linguistic approximation, namely, the maximum depth of the nested hedges (C option) and the threshold value of the evaluation function for approximation (T option). The PARA instruction in Example 2 states that the maximum depth of nested hedges is 2 and the threshold value is 0.12. We can omit PARA instruction in AR Block, and in that case the default values are assumed.

The method of linguistic approximation and the evaluation function for linguistic approximation will be discussed at the end of this section.

For the implementation of AR System, since the construction and manipulation of fuzzy sets and fuzzy relations are processed in FSTDS System, the processings in AR System are the following:

(i) Processing for the definitions of universes of discourse and primary terms: AR System passes to FSTDS System the sets and fuzzy sets defined in the Universe of Discourse Block and Primary Term Block. Since the definitions of universes of discourse and primary terms are made fit to FSTDSL syntax, the only thing AR System has to do is to add operators SET and FSET when they are omitted.

(ii) Processing for composite terms and conditional propositions: A composite term is translated into an expression in FSTDSL, that is, hedges are replaced by the corresponding operators in FSTDSL using (2.9)-(2.16) and connectives are translated into prefix operators. For example, the proposition:

```
P: X IS NOT (VERY SMALL OR SORT_OF MIDDLE);
```

is translated into an FSTDSL statement:

```
P := ADIF(U, UNION(CON(SMALL), NORM(INTERSECTION(ADIF(U,CON(CON(MIDDLE))),
                                          DIL(MIDDLE)))) );
```

A conditional proposition IF ... THEN ... is translated by either (2.3) or (2.4) under user's specification.

(iii) Approximate reasoning from the specified propositions: If the proposition P1 involves a object and P2 two objects, then AR System passes the FSTDSL statement:

CONSEQUENCE := IMAGE(P2,P1);

to FSTDS System. It should be noted that $A \circ R$ in (2.1) reduces to the image of A under R since A is a unary fuzzy relation, i.e., an ordinary fuzzy set.

(iv) Linguistic approximation: The part of linguistic approximation takes the most CPU time in AR System.

We shall discuss the method of linguistic approximation in our system.

Assume that B is the fuzzy set obtained by approximate reasoning and $t_1, t_2, \ldots, t_m$ are primary terms for linguistic approximation. If we approximate B using n depth of nested hedges which are chosen from hedges $h_1, h_2, \ldots, h_k$, and one of the primary terms $t_1, t_2, \ldots, t_m$, then in order to find the best approximation we must generate

$$(1 + k + k^2 + \cdots + k^n)\cdot m \tag{3.7}$$

composite terms. Since the number m and n can be specified by a user and k is 9 (more exactly, 3 definitions of the hedge *slightly* and two of *rather* make $k = 12$) in our system, the number of composite terms to be generated is so large, say, 814359 in Example 2 because $m=3$, $n=5$ and $k=12$, that we could not use the really best linguistic approximation.

Therefore we use the following strategy.

First, we apply each hedge to given m primary terms and compare its result with the consequence fuzzy set B. If we obtain a sufficient approximation, that is, the value of evaluation function is less than or equal to the threshold value, then it is accepted as the linguistic approximation of B. Otherwise we choose one composite term of the best approximation in the first linguistic approximation and apply each hedge again to it. If we cannot still obtain a good one, the same is repeated until n times. If a sufficient approximation has not been obtained yet after n times of repetitions, the best approximation by this time would be considered as a linguistic approximation of B.

This method generates only

$$m\cdot(1 + k) + (n - 1)\cdot k \tag{3.8}$$

composite terms even in the maximum case, e.g., it is only 87 on the same condition as before. There is, however, no guarantee for the really best approximation.

For the evaluation function, there might be several definitions. We have adopted the evaluation function such as

$$f(B,B') = \frac{\sum |\mu_B(u) - \mu_{B'}(u)|}{\#(U)} \tag{3.9}$$

where B is the fuzzy set obtained by approximate reasoning, B' the fuzzy set modified by hedges in linguistic approximation and U a universe of discourse of B and B', and $\#(U)$ denotes the number of elements in U, $|x|$ the absolute value of real number x and Σ the ordinary sum over U. This evaluation function means an average of the absolute difference values of the grades of B and B'. It is also possible to represent f using operators in FSTDSL as

$$f(B,B') = \frac{CD(ADIF(B,B'))}{\#(U)} \tag{3.10}$$

where the operator CD, ADIF and # denote the cardinarity, the absolute difference and the the number of elements, respectively.

We shall conclude this section with another example of description and result in full in AR System.

[Example 3] We give a description in AR System in Fig.2 and some of results in Fig.3 and Fig.4. In Fig.3 a sufficient linguistic approximation is obtained but not in Fig.4. Really, the propositions in Fig.3 are the same as those in Example 2. It should be noted that we choose the arithmetic rule for conditional propositions (2.3) rather than the maximin rule for conditional propositions (2.4).

```
          APPROXIMATE REASONING SYSTEM SOURCE LIST

*************************** UNIVERSE OF DISCOURSE BLOCK ****************
       U5=(1,2,3,4,5) ;
       U6=CP(U5,U5) ;
*************************** PRIMARY TERM BLOCK *************************
 /U5/ SMALL=(1/1,.8/2,.5/3,.2/4) ;
 /U5/ MIDDLE=(.2/1,.5/2,1/3,.5/4,.2/5) ;
 /U5/ LARGE=(.2/2,.5/3,.8/4,1/5) ;
 /U6/ APPROXIMATELY_EQUAL=(1/<1,1>,1/<2,2>,1/<3,3>,1/<4,4>,1/<5,5>,
    .8/<1,2>,.8/<2,1>,.8/<2,3>,.8/<3,2>,.8/<3,4>,.8/<4,3>,.8/<4,5>,
    .8/<5,4>,.4/<1,3>,.4/<3,1>,.4/<2,4>,.4/<4,2>,.4/<3,5>,.4/<5,3>);
*************************** PROPOSITION BLOCK **************************
PROP1:A IS LARGE;
PROP6: A AND B ARE VERY APPROXIMATELY_EQUAL ;
PROP8:A IS MORE_OR_LESS SMALL ;
PROP9:IF A IS VERY LARGE THEN B IS NOT LARGE ;
PROP10: A IS MIDDLE ;
PROP11: A AND B ARE MORE_OR_LESS APPROXIMATELY_EQUAL ;
PROP12: A IS PRETTY LARGE ;
PROP14: IF A IS PRETTY LARGE THEN B IS RATHER LARGE ;
*************************** AR(APPROXIMATE REASONING) BLOCK ************
       PARA(C=5,T=13) ;
       AR(PROP10,PROP14/MIDDLE) ;
       AR(PROP8,PROP6/SMALL LARGE MIDDLE) ;
       AR(PROP12,PROP11/SMALL LARGE MIDDLE) ;
       AR(PROP1,PROP9/SMALL MIDDLE LARGE) ;
       AR(PROP10,PROP9/SMALL MIDDLE LARGE) ;
*************************** END OF APPROXIMATE REASONING ***************
```

Fig.2. An example of description in Approximate Reasoning System.

```
-----> AR(PROP1,PROP9/SMALL MIDDLE LARGE) ;

..... OBTAINED SUFFICIENT LINGUISTIC APPROXIMATION

* LINGUISTIC APPROXIMATION...  MINUS MORE OR LESS SMALL

* PRIMARY TERM...  SMALL

* FUZZY SET WHICH IS PRIMARY TERM...
     FSET(1/1, 0.8/2, 0.5/3, 0.2/4);

* FUZZY SET GENERATED BY COMPOSITIONAL RULE OF INFERENCE...
     FSET(1/1, 0.96/2, 0.8/3, 0.56/4, 0.5/5);

* FUZZY SET MODIFIED BY SOME HEDGES...
     FSET(1/1, 0.9197/2, 0.771/3, 0.5468/4);

* EVALUATION FUNCTION VALUE...  0.1165

* THRESHOLD VALUE...  0.130

* COUNTS OF ADDING THE LINGUISTIC HEDGES...  5
```

Fig.3. A result of Approximate Reasoning System (1).

```
-----> AR(PROP12,PROP11/SMALL LARGE MIDDLE) ;

..... NOT OBTAINED SUFFICIENT LINGUISTIC APPROXIMATION
      FOLLOWING RESULTS ARE THE BEST APPROXIMATION IN THIS TRIAL

* LINGUISTIC APPROXIMATION...  MINUS PRETTY MORE OR LESS MIDDLE

* PRIMARY TERM...  MIDDLE

* FUZZY SET WHICH IS PRIMARY TERM...
     FSET(0.2/1, 0.5/2, 1/3, 0.5/4, 0.2/5);

* FUZZY SET GENERATED BY COMPOSITIONAL RULE OF INFERENCE...
     FSET(0.5/1, 0.5/2, 0.5/3, 0.5/4, 0.5/5);

* FUZZY SET MODIFIED BY SOME HEDGES...
     FSET(0.5027/1, 0.5947/2, 0.5947/4, 0.5027/5);

* EVALUATION FUNCTION VALUE...  0.1390

* THRESHOLD VALUE...  0.130

* COUNTS OF ADDING THE LINGUISTIC HEDGES...  5
```

Fig.4. A result of Approximate Reasoning System (2).

4. Conclusion

We have described Approximate Reasoning System which facilitates the definition of universes of discourse on which primary terms are defined, the description of propositions and the execution of approximate reasoning and linguistic approximation.

For the implementation of AR System, the translation of the description in AR System to FSTDSL is programmed in PL/I and the construction and manipulation of fuzzy sets and fuzzy relations are programmed in FSTDSL/FORTRAN, and it is currently running on a FACOM 230-45S computer. The current version of AR System is an interpreter version.

AR System cannot infer from the propositions involving n objects ($n \geq 3$). The compositional rule of inference may be applied to that case. This limitation of our system owes the lack of implementation of the composition of m-ary ($m \geq 3$) and n-ary ($n \geq 3$) relations in FSTDS System.

Zadeh [1] proposed the formulation of the approximate reasoning from the propositions with truth-quantifier, i.e., in symbols:

$$\begin{array}{l} P_1\text{: (x ia A) is } \tau_1. \\ \underline{P_2\text{: (x and y are R) is } \tau_2.} \\ P_3\text{: (y is B) is } \tau_3. \end{array} \qquad (4.1)$$

where x and y are object names, A, R and B are a fuzzy set, a fuzzy relation and a fuzzy set, respectively, and τ_1, τ_2 and τ_3 are linguistic truth values. This facility is now being implemented in AR System.

For the interpretation of conditional propositions, Zadeh's formulation does not always provide the consequence which fits our intuitions. Since the improved methods are proposed which give the consequence which fits our intuitions under several criteria [8], we have the plan to implement these formulations.

For linguistic approximation, at present there is no simple or general method for finding a good linguistic approximation. The studies of efficient algorithms for better linguistic approximation may be needed.

As a basis of question/answering systems and intelligent accesses to data base systems, approximate reasoning plays so important role in the interface between such systems and users that AR System will find various applications.

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